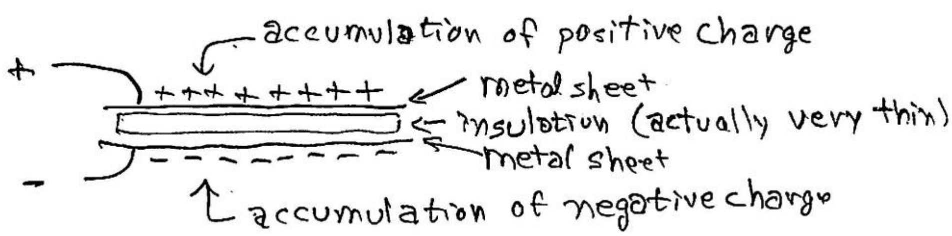
Learning from a demonstration of an electrical spark vaporizing a bit of copper wire

After we began talking about basic electronics, with charge, voltage, current, resistors, and LEDs, I mentioned capacitors. There was one demo in the Electronics Playground with a modest capacitor (100 microfarads or µF). I brought out a huge capacitor that our son, David, bought with his own money when he was about 10 years old. I told him that we could make powerful sparks with it, and that sold him on it. It's rated at 25,000 µF at 30 volts.

Capacitors store electrical charge. They gain in voltage (V) as the charge (Q) increases, $V = QC$. Here, C is the capacitance. This represents a storage of energy. For a bit of charge, dQ, transferred at a voltage V, the energy increment is $dE = V\,dQ$. Skipping a bit of simple calculus, we can just note that the average bit of charge is transferred at the average of 0 and final voltage, V, so that the total energy stored is $E = VQ/2$. We can also write that as $V^2C/2$. So, a larger capacitance matters, and, more so, a higher voltage tolerance.

Capacitors are made of thin sheets of conductive metal separated by thin layers of insulating material. The thinner, the better. There are two sets of sheets. One is connected to the terminal where positive voltage is applied, the other to the negative terminal. Across two sheets of opposite polarity the opposite charges accumulate. The charges are kept from jumping across by the thin insulating layer.



Between any of these sheets there is a very strong electric field – that is, the difference in voltage divided by the distance. That's the origin of the limit on voltage. Above a certain electric field the insulation fails and the charges flow across the damaged insulation – and quickly, at huge rates of flow (electric currents). That permanently damages the capacitor.

You can make your own capacitor, if not efficiently, by layering sheets of metal foil and an insulator such as oiled paper, then rolling them up. Don't forget to attach two wire leads for connecting them. David's huge capacitor is that structure, with the insulation made "on site," as I'll explain. Its metal sheets are likely aluminum. More expensive capacitors are made with tantalum metal; look up its interesting properties. Re the insulation: it starts as a liquid mixture that creates insulating "corrosion" on the plates when electric current is run through it. It stops reacting and becomes an insulator. In any event, don't run an operating current into the capacitor the wrong way, positive voltage into the negative leads. The reaction will create gas that will ultimately explode the capacitor.

A good capacitor has very low, very slow leakage of charges between the plates internally. We tested this on David's capacitor. The measured voltage changed from, say, 14.00 V to 13.98 V over minutes.

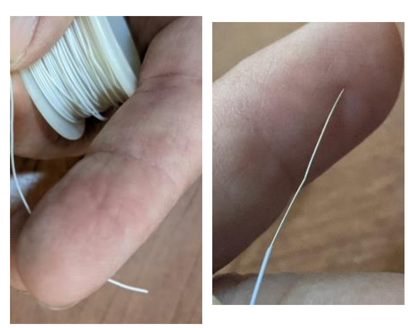
On the other hand, once connected to an external circuit, a good capacitor can release its charge at huge rates. It's why we'll get a pulse of current that will melt or vaporize the small wire. Truly huge capacitors are used on loading docks to power electric motors for cranes to lift cargo onto ships. They get partly recharged from the energy of dropping back down to dock level via a generator. The capacitors have an advantage over batteries or feeding power from power lines. They can put out huge currents for a fast lift that would damage batteries of similar capacity.

However, capacitors don't store much energy for their size. You won't find electric cars run on capacitors that would get recharged at power stations like those for Tesla cars. It's just very hard to store electricity other than chemically. As we'll see shortly, that big capacitor at 15V stores about 3 joules of energy. Compare that to a small AAA battery, which is rated at 1.5 volts and can provide about 600 milliampere hours of current. One way to look at that is it can run for 3600 seconds at a current of 0.6 amperes. That's a transfer of 0.6*3600 = 2160 coulombs. Multiply that by the voltage to get 3240 joules! On the other hand David's capacitor can provide hundreds of amperes of current. Capacitors are termed power-dense but poor at energy density. Power is the *rate* of providing energy. Capacitors' overwhelming use is as small ones in electronic circuits that handle alternating currents for precise control of timing (frequency) or of filtering (preferentially allowing higher or lower frequencies of signals).

Now to our demo. We're going to charge the capacitor and then connect it across the tiny wire to heat that wire extremely rapidly. We have to use some connecting wires, or leads, from the capacitor to the wire. We have to ensure that the power is delivered almost all to the tiny wire and not used up in the connecting leads.

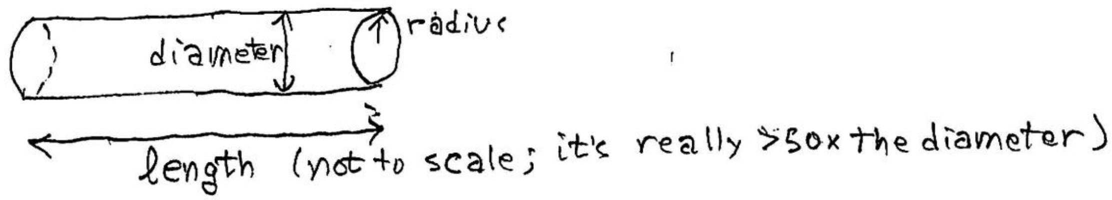
Let's figure out the energy that can be stored in the capacitor. Then we'll figure out how much energy it takes to melt or vaporize the mass of copper in the tiny wire. The stored energy is the square of the voltage multiplied by the capacitance and divided by two, or $V^2C/2$. Let's plug in those numbers. We'll use metric-system units so that everything comes out automatically with no conversion factors. The capacitance is 25000 μF. Since a micro-anything is a millionth, we have 25000 millionths of a farad, or 0.025 F. We get

$$E = \frac{V^2 C}{2} = \frac{(15)^2 (0.025)}{2}$$
$$= 2.8J$$



Now let's get to the wire. I chose what's called wire-wrap wire because it's thin, with low enough mass that 2.8 joules can melt it or vaporize some of it. The wire meets a national standard for thickness. Ion particular it is 30-gauge wire. It has a diameter of 0.256 mm or close to 1/100 inch in ancient units. We need to find its volume, then its mass. I'll tell you that I chose a short length of wire so that its mass is low enough to be melted/vaporized by the amount of energy in the capacitor. By the way, previous LCA students and I built the complicated circuitry that runs our light-up periodic table of the chemical elements, using about 400 segments of wire-wrap wire and many standard electronic and structural parts. The story is online at https://science-technology-society.com/stem-work/#lupt.

The volume of a cylinder (it is a tiny cylinder) is the area of its cross-section multiplied by its length



The area of a cross-section is *pi* (π) multiplied by the square of the radius. The radius in half the diameter, or 0.128 mm. Let's get everything into units at the base of the SI or metric system. That will be meters for sizes. We'll rewrite 0.128 mm as $1.28x10^{-1}$ millimeters (mm) and then as $1.28x10^{-4}$m. The length I chose is 1.5 cm, or a bit over half an inch in units long obsolete in science. Rewrite that as $1.5x10^{-2}$m. We get this volume:

$$V = \pi r^2 L = (3.14)(1.28x10^{-4})^2(1.5x10^{-2})m^3$$
$$= 7.7x10^{-10}\, m^3$$

Here I've gone to exponential notation. This may take some getting used to, though it's very, very common in science. I added an appendix to get you some background in this.

Now we can figure out its mass. The density of copper is nearly 9 times greater than that of water. Now recall what you might know about water. One liter of water weighs one kilogram, within tiny offsets that depend on, say, the temperature. A liter is 1000 cubic centimeters in units that people often use. We can also say that it's 1000 x $(0.01)^3$ cubic meters or 1/1000 of a cubic meter. So, a cubic meter of water "weighs" (has a mass of) 1000 kilograms. A cubic meter of copper has a mass near 9000 kg. Let's multiply our thin wire's volume by this density:

$$Mass = m = volume\ x\ density$$
$$= (7.7x10^{-10}m^3)(9x10^3 kg\, m^{-3})$$
$$= 6.9x10^{-6}\, kg$$

Pretty tiny, about 7 milligrams; it's just below the 10 mg that's weighable on our little balance (which is really a scale, not a mass balance, as we discussed a while ago).

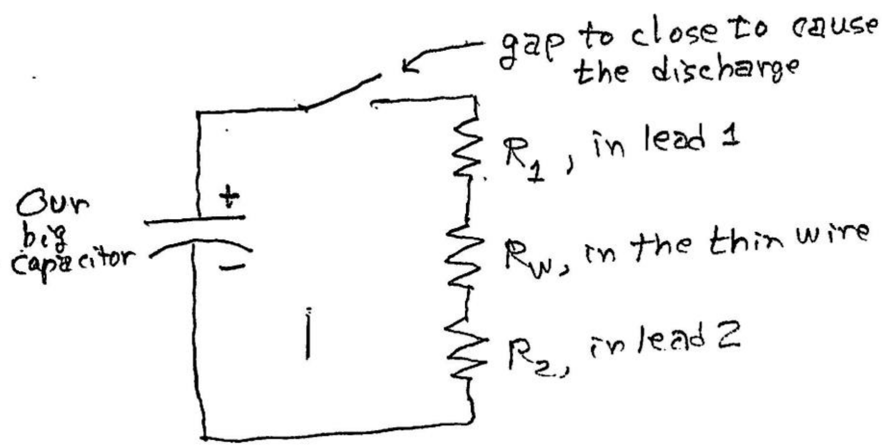
We can now figure out how much energy it takes to melt that mass of copper. We can also figure out the energy needed to vaporize that mass of copper. A simple search reveals the value of the *heat of fusion* (melting) of copper from its standard state near room temperature to a liquid just at its melting point, which is 1085°C (1985°F, so much higher than the temperature of an oven broiler). The heat of fusion is 285 kilojoules per kilogram, or 285,000 joules per kg. Multiply that by the mass just calculated earlier and we get 2.0J. Ah, this will work.

When we do the experiment, it looks like some of the metal actually vaporizes. It's easy to show that there's far too little energy to vaporize all the metal. We can look up the heat of vaporization of copper. It's 322 kJ per mole. A mole of copper atoms is 63.5 grams or 0.0635 kg. We can convert the heat of vaporization to kJ per kg, or, a bit more transparently, we can convert the mass of copper in the wire to moles:

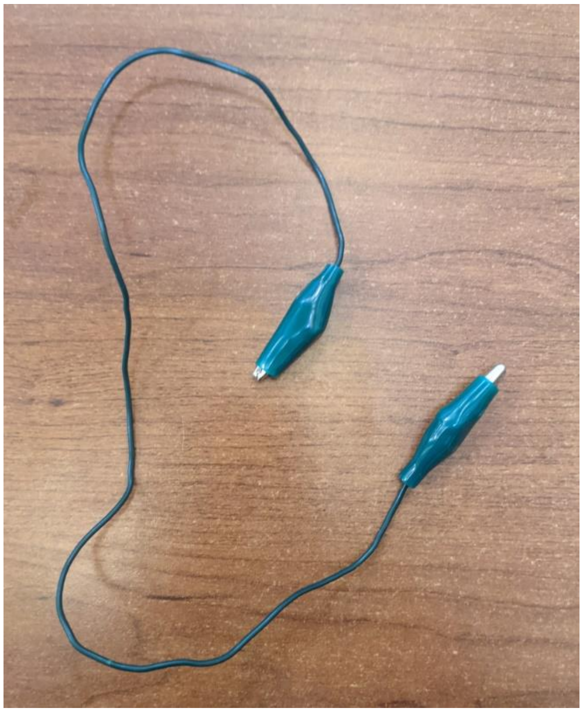
$$
\begin{aligned}
n &= number\ of\ moles \\
&= (mass)/(mass\ per\ mole) \\
&= \frac{6.9x10^{-6}\,kg}{0.0635\,kg\ mol^{-1})} \\
&= 1.09x10^{-4}\ mol
\end{aligned}
$$

Multiply that by 322,000 J per mol ("mole" and "moles" become just "mol" in notation) to get 35 joules! By the way, the temperature of copper as a vapor, at its minimum, is 2,562°C or 4,644°F, about halfway to the temperature of the Sun's surface.

OK, we know what to expect. We have two tasks remaining. One is to charge the capacitor safely (for itself and for the charging device). The other is to figure out how to connect the capacitor to the tiny wire with a low resistance in the connections, so that the energy gets dissipated mainly in the wire and not in the connectors. Let's do the latter first. Here's a sketch with the capacitor ready to connect to the wire through two leads. All three pieces have their own electrical resistances, as noted:

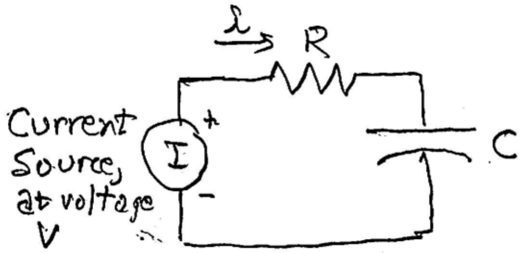


We learned earlier that a current *i* flowing through a resistor *R* creates a voltage $V = iR$ and a power *P* that is voltage times current. We have $P=i^2R$. What matters most is that, in a string or resistances (incoming lead with $R=R_1$, tiny wire with $R=R_w$, outgoing lead with $R=R_2$), the power divides in proportion to the resistances. While it's tempting to use handy alligator clip wires or leads (below) to connect to the tiny wire, that won't work. The alligator leads have wires not much thicker than the tiny wire, so their resistance per unit length is about 1/6 that of the tiny wire. However, they are each about 30 cm long, for a total of 60 cm, 40 times greater than the tiny wire. Most of the energy would go into the leads. It's worse that that. The wire resistance in the leads (22-gauge wire) is only 0.03 ohms (0.03 Ω each, but the resistance measured with a multimeter is 0.6 Ω. The real resistance is in the solder connections from their internal wires to the alligator clips. If the capacitor discharge is going to melt anything, it will be the solder joints!

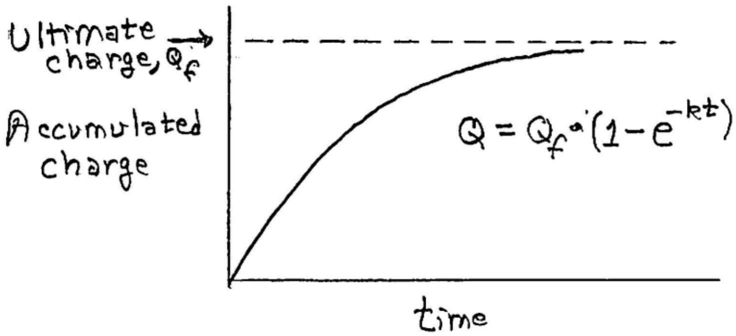


So, we will use massive jumper cables from our car.

On to how we'll charge the capacitor. This needs some care. We can charge fast with a high-current power supply or slowly with a low-current power supply. In any case, we have to have a resistor in the charging circuit that limits the current to what the power supply can deliver



Let's consider the monster heavy-duty power supply. It can put out 15V at up to 30 amperes. If we connect it straight to the capacitor without a current-limiting resistor that will trip the electronic circuit breaker in the power supply. We can choose a small-value resistor, say, 1 Ω, and get a current of 15A. However, we look at the power dissipated in the resistor, i^2R, and find that it's 225 watts! My tiny resistors are rated at 1/8 or ¼ watt! A resistor can stand a temporary overload, but let's just make it simple. I have resistors rated at 5W dissipation and 100 Ω. That gives a peak current at the start of charging (when the capacitor is at zero volts) or 0.15A. The power dissipation is then 2.25W. We're fine. How long will it take to charge the capacitor to 15V? First, we figure out the total amount of charge, Q, that has to be transferred. That's Q = VC, which turns out to be 0.375 coulombs. The charge is equal to the current multiplied by the time. If we had the initial current maintained during charging, the charge transferred is Q = it, where *t* is the time. We get t = Q/i, which is 0.375/0.15 or 2.5 seconds. It takes longer, because the voltage building up in the capacitor opposes the voltage of the power supply. The trend is a saturating curve



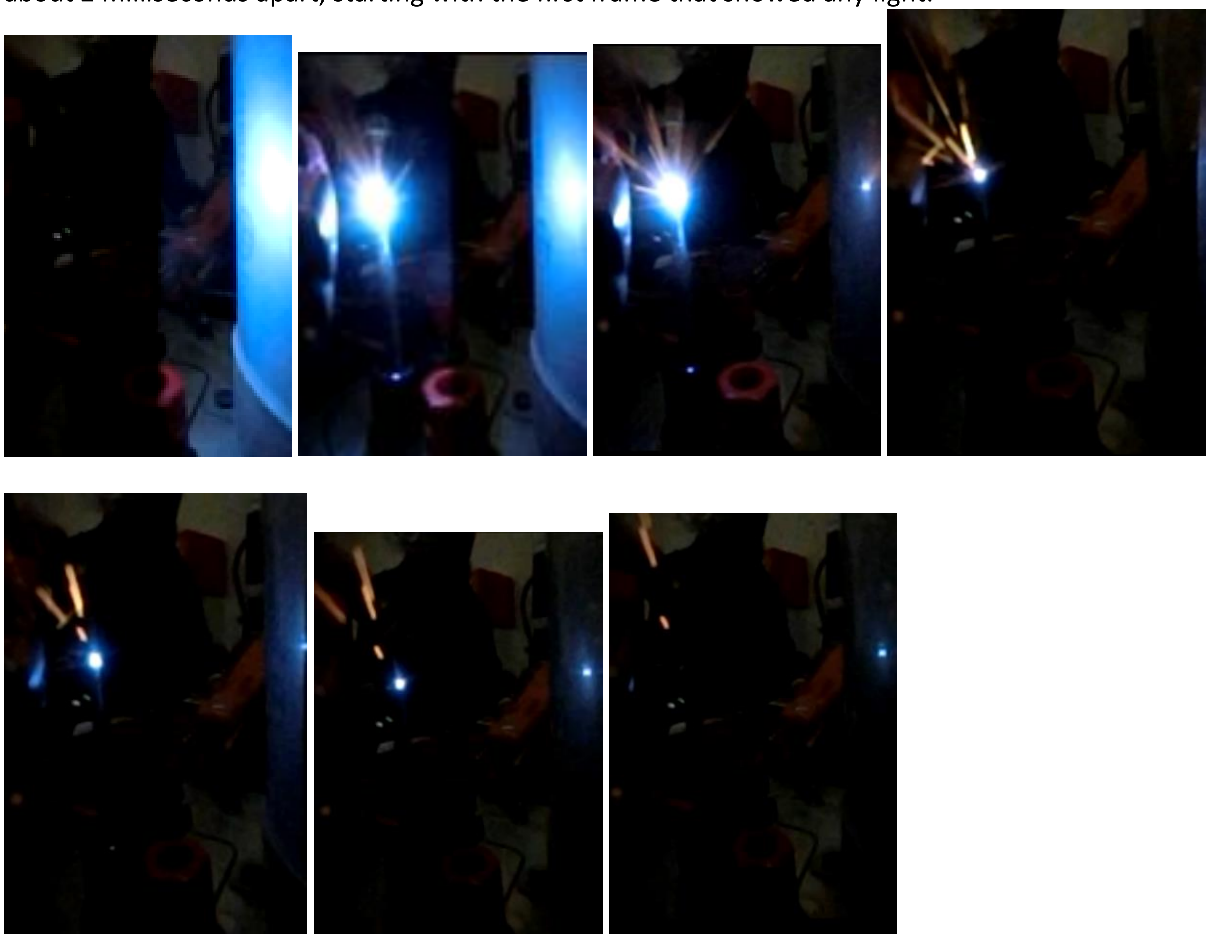
We might wait about 3 times as long to be sure.

We could also use the electronic "workbench" at 15V (or even 30V, using positive to negative supplies). It can probably put out a maximal current of, say, 50 milliamperes (mA). The current is 1/3 as large, so the charging time is 3x longer. We need a different current-limiting resistor here. To get 1/3 as much current as with the big power supply at the same voltage, we need a resistor that's 3x bigger, or 300 Ω.

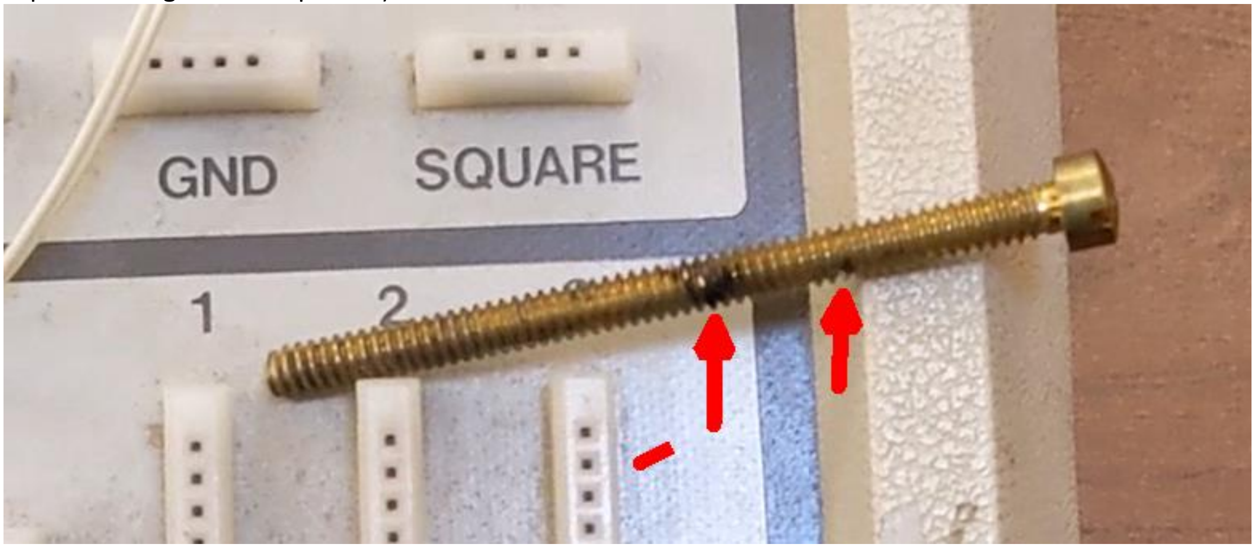
Now to the demo, at last. We saw the spark and we saw about half the length of the wire disappear. Dramatic. Let's learn more. Let's take high-speed videos with Lou Ellen's Casio Exilim camera, a really handy camera that can take up to 1000 frames per second! the resolution is low at that speed. We'll see if we have to compromise.

The full results will be put into this document when we get that video!
Meanwhile, there's some interest in the final results. First, only half the wire disappeared – it partly melted (where did that end up?) and it partly vaporized. Second, almost all the energy in the capacitor was used up. Isaac measured the voltage remaining on the capacitor and it was only 0.3V. Question: How did the current continue to flow when the wire was disappearing? We may get some answers with the high-speed video.

Note: we could charge the capacitor to its full 30V using the electronics workbench. That has two power supplies, +15V and -15V. We can hook the positive end of the +15V supply to the capacitor's + terminal and the negative end of the -15V supply to the capacitor's negative terminal. We get 30V across the capacitor. What happens to the wire with *four* times the energy applied? Recall that the energy is proportional to the square of the voltage, which we doubled. The result was an even more dramatic spark. We took a video at 480 frames per second. The resolution is low necessarily (pixels are lumped together to save processing time) but the video shows interesting features. Here are 7 frames, each about 2 milliseconds apart, starting with the first frame that showed any light:



Now for some interpretation. Let's not pay attention to the darkness of the images; the light in the classroom was rather low for video. Nonetheless the flashes are clear The first image shows the greatest light intensity. The light is coming (alas) from behind the brass machine screw that we used as a contact. The second image shows the light coming around the screw and still intense. The third image shows molten copper being thrown out, as do all the succeeding images. The velocity is not great. Since the image height at the distance of the contact post is about 6 cm, it appears that the tracks move at about 1.2 cm in 4 ms (milliseconds). If we're tracking the right objects, that's a speed of 3 m s^{-1} (3 meters per second), around 7 mph. Now, at least in frames 3 onward, the copper wire has disintegrated. All of the energy would have been delivered in no more than 4 ms. That's 11 J (close to four times 2.8 J, given twice the voltage) in that time, so the power as energy over time is 11 /0.004 or 2750 watts! the peak power would be about twice the average power or 5500W. The peak current would be this wattage divided by 30V, or over 180 amperes. One last trace of the high instantanous power is the “instantaneous” obliteration of part of the machine screw where the wire touched it (two separate firings of the capacitor):



This turned out be an exercise in a little drama, plus understanding power and energy, learning energy balance, looking at a phase transition from metal to liquid and vapor, calculations in geometry to figure out wire mass, using the metric system, using handy exponential notation, understanding how voltage partitions itself along the parts of a circuit (the leads and the wire)…and capturing the dynamics with high-speed video, to see what happens where. It’s a great help that Lou Ellen, David, and I have the background in diverse sciences, plus equipment and methods of measurement acquired over the years. We’re pleased to share it all with you.

Here’s most of the team – Erica, Samantha, and Isaac. Jules and Dov left for the afternoon break as we set up the re-take.

Appendix: exponential notation

This is very handy. We can avoid really long decimal notation. Consider the calculation of the volume of the tiny wire. Its cross-sectional area is 1.28 mm, squared, multiplied by π. That's 3.14 x 0.000128 x 0.000128 square mm (mm^2) or 0.000000514 m^2 (square meters). It's really easy to lose count of the zeroes. Consider also writing the number of atoms in a mole of an element; this is Avogadro's number. That's 602,200,000,000,000,000,000,000. No one wants to keep writing that, either.

We then take what we might say are the significant figures (not the same use as in statistical analysis, but close) and then figure out how many decimal places we move left or right. We can write 0.000128 in two pieces. One is 1.28, the "numerical" part, taking only the first nonzero digit, the "1," and adding the remaining digits after the decimal point. The other part is a factor that says how many tens-places we moved forward. We actually moved backward 4 places, so we write $1.28x10^{-4}$; we moved 4 places backward. Similarly, Avogadro's number has "interesting" number 6.022 and a move forward by 23 places. We write is as $6.022x10^{23}$.

We do the same with names of units. If the multiply meters times meters, we write m^3 instead of m x m x m or mmm (very confusing) or "meters cubed" (very hard to figure out how it combines later with "per square meter" or the like). There are inverse units, too. Density as mass per cubic meter is 1 over meters cubed. We write it as m^{-3}.