Simple calculations of the speed of Low Earth Orbit:
I'll use energy conservation as the most undemanding case (lowest v_{orb}).

Low Earth orbit is stable when the satellite has a velocity, v_{orb}, high enough to create a centrifugal force, mv_{orb}^2/r, that balances the gravitational force, GMm/r^2. So, we have

$$\frac{m{v_{orb}}^2}{r} = mg(r)$$

We can cancel out the mass, m, since it's the same of both sides.
Here, m is the mass of the satellite, r is the orbital diameter measured from the center of the Earth, G is the universal gravitational constant, and M is the mass of the Earth. We can get rid of G and M by using $GM/r_0^2 = g_0$, the gravitational acceleration at the surface of the earth, where r_0 is the distance of the Earth's surface from its center – the radius of the Earth, which is 6368 km, on average (less at the poles, more at the equator).

We're ready to figure out what the gravitational acceleration is at the distance of the satellite. It's less than at the Earth's surface; it falls off as $1/r^2$, so we have

$$g(r) = g0\frac{{r_0}^2}{r^2}$$

Now we can figure out the orbital velocity in terms of the gravitational acceleration at the Earth's surface (close to 9.8 meters per second squared):

$$\frac{{v_{orb}}^2}{r} = g0\frac{{r_0}^2}{r^2}$$

$$or$$

$${v_{orb}}^2 = g_0\frac{{r_0}^2}{r}$$

Plug in the values $g_0 = 9.8$ m s^{-2}, $r_0 = 6.37\text{x}10^6$m, and $r = 6.67\text{x}10^6$ m, and we get the orbital velocity

$$\begin{aligned} v_{orb} &= 7660\, m/s \\ &= 4.78\, mi/s \\ &= 17,200\, mph \end{aligned}$$

- The needed speed is higher than that, for two reasons:
  - 1) Speed is lost in rising to altitude, from energy conservation; you need about 5% higher speed. Again by energy conservation, the work done against gravity is readily calculated as the gravitational potential energy at orbit minus the gravitational potential at the surface:

$$\begin{aligned} \Delta U &= \frac{GMm}{r_{orb}} - \frac{GMm}{r_0} \\ &= \frac{GMm}{r_0}\left(\frac{r_0}{r_{orb}} - 1\right) \\ &\approx 0.05\frac{GMm}{r_0} \end{aligned}$$

    The last line uses the fact that radius of LEO is about 5% larger than the radius at the Earth's surface. Here, G is the universal gravitational constant, M is the mass of the earth, and m is the mass of the launched object. I'll put in a subscript to make that m_{obj}, to avoid confusion for *m* as meters. We can get rid of G and M using the relation that expresses the gravitational acceleration at the Earth's surface, g_0:

$$\frac{GM}{r_0^{\,2}} = g_0$$

We get

$$\begin{aligned}\Delta U &= 0.05\, m_{obj}\, g_0 r_0 \\ &= 0.05\, m_{obj}\; 9.8\, m\, s^{-2}\, 6.3x10^6\, m \\ &= 3.1x10^6\; m^2 s^{-2}\; m_{obj}\end{aligned}$$

We have to add this gain in potential energy to the gain in kinetic energy, ½ $m_{obj}v_{orb}^2$. If all the energy starts as kinetic at a launch speed v_{launch}, we have (canceling out m_{obj} all through,

$$\frac{1}{2} v_{launch}^{\;2} = \frac{1}{2} v_{orb}^{\;2} + \Delta U / m_{obj}$$

Putting in v_{orb} = 7660 m s^{-1}, we find that the ΔU term is about 10% of the kinetic energy of orbiting, so that v_{launch} is another 5% higher than v_{orb}.

  - and, (2) a significant amount of speed is lost in traversing the atmosphere. I haven't bothered with this but I'd be quite surprised if it doesn't require another 5-10% at a minimum.
  - and, (3) thrust vectoring to change from a vertical trajectory to a tangential trajectory means that some work has to be done perpendicular to the trajectory at times; this uses fuel but doesn't contribute to the orbital energy.
- OK, if we say that about 20% higher v_{launch} is needed, as a rough estimate, then we're up to 1.2*7660 m s^{-1} or about 9190 m s^{-1}. To get that in a single stage with a solid propellant having v_{ex} of 2500 m s^{-1}, we need a higher burn ratio: $\ln(m_0/m_f)$ = 9190/2500 = 3.68. That means m_0 is almost 40 times the mass of the shell and the payload. This is unachievable. → One needs two stages, with all the separation mechanisms that won't stand 3500 g!