How cold is that rain?

If you've stood outside in the rain, even on a warm day, you've felt the chill of the rain. There are two reasons, the first being that water takes up heat from your skin much faster than does air when both the rain and the air are at the same temperature that's lower than body temperature – the same reason that metal feels cold even when it's at room temperature or even above. The second reason is that the rain *is* at a lower temperature than the air, and we can estimate just how much cooler it is, based on air temperature and relative humidity. We can also reverse the calculation, to estimate relative humidity from the temperatures of the air and of the rain (catch the rain in an insulated cup).

I'll get to a quantitative account shortly. First, the qualitative story: as the raindrop falls, water evaporates from its surface as vapor, carrying away heat and cooling it. However, as it cools, it achieves a certain rate of heat absorption from the air that's warmer than it is. Also, as it cools its own vapor pressure drops. The drop reaches a quasi-steady state in temperature when the two processes balance, cooling by evaporation and heating by convective heat transfer from the air. We say that the raindrop reaches the "wet bulb" temperature, referring to the old sling psychrometers, used in meteorology in the past, or even today in some areas and for demonstration of physical principles.

You can skip to using the theory to estimate the rain's temperature from the air temperature, relative humidity, and air pressure. I created a spreadsheet, in which you can enter the values in either metric or English units. There is a second section to it, in which you can do the inverse, calculating the relative humidity from the air temperature, rain temperature, and air pressure.

Theory

(For those inclined to math and physics, I have a short derivation posted, though it misses the factor 1.1 that appear later here)

We need to formulate the rate of cooling, per unit area, as it depends upon the temperature of the raindrop itself, the amount of moisture in the air, and two things that determine the rate at which water vapor can leave the surface of the raindrop. The last two factors are the speed of the wind past the raindrop (its rate of fall) and the size of the raindrop. They determine what's called the boundary-layer conductance for water vapor, which we'll give the symbol g_{bw}, with "b" for boundary and "w" for water vapor. Calculating g_{bw} can be tricky, but it doesn't matter! Both the cooling from water evaporating and the heating from the surrounding air depend upon the boundary-layer conductance, so it becomes a factor that cancels out. The important thing is that this conductance be rather high, so that the raindrop reaches its steady state with little delay.

Relation to evaporation rate and heat of vaporization of water: The cooling rate will be the rate at which water vapor is leaving (again, per area), which we'll denote by *E*, multiplied by the amount of heat absorbed to evaporate a given amount of water – its heat of vaporization, which we'll denote by λ. Then we have for a cooling rate, *C*,

$$C = \lambda E$$

We should be careful about units. It's convenient specify *E* in units of moles of water (18 g) per area (and area in, say, square meters, to stay "metric"). Then we'll also use λ in heat of vaporization per mole; if our unit for energy (heat) is joules, then

$$\lambda = 44,000 \, J / mol$$

(This is approximate, since the heat of vaporization changes with the temperature of the water, but not very much over, say, 10°C.)

How the evaporation rate depends on conditions in the air: Now, an accurate way to express *E* is as the boundary-layer conductance, multiplied by the difference in water vapor content – that at the surface of the raindrop, minus that in the surrounding air. I'll assume that we'll use g_{gw} in "molar units, moles per square meter per second, as is common in metric engineering and science and especially in plant physiology and ecology (where I've done a lot of my work). Then, the appropriate measure of water vapor content is the fraction of the air that is water vapor, as moles – the so-called mole fraction. The fraction of water vapor in air is simply the vapor pressure of water divided by the total air pressure, because air and water act very closely as being independent of each other, as "ideal gases." We'll discuss vapor pressure right away, after writing the equation for evaporation rate:

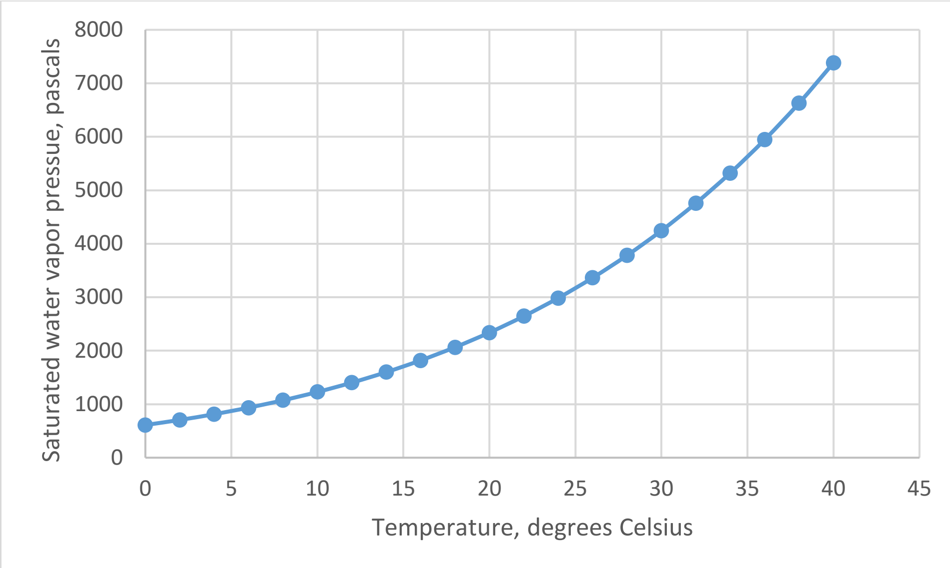
$$E = g_{bw}\left(\frac{e_{drop} - e_{air}}{P}\right)$$

Here, e_{drop} is the water vapor pressure at the surface of the raindrop and e_{air} is the water vapor pressure in the air, while *P* is the total air pressure.

Now we need to discuss water vapor pressure, *e*. First, it is a well-defined, measurable quantity. Second, from the surface of pure water it has a value that is determined only by temperature; it achieves the saturating vapor pressure, e_{sat}. Murray, in 1967, examined all the measured values of e_{sat} over many temperatures and came up with an equation to express it accurately:

$$e_{sat}(T) = 610.8 Pa \exp\left(\frac{17.269T}{237.2 + T}\right)$$

The results are expressed in the metric (SI) unit of pressure, the pascal. To give you a feel for this, normal air pressure at sea lelel is 101,300 Pa. If the temperature is 30°C, then e_{sat} is 4246 Pa, so that 4246/101,300 = about 4% of the molecules in the air are water. If you're not familiar with the exponential, you can simply look at the results later, or use the spreadsheet that I provide separately to calculate the drop in temperature for any conditions. Note also that I'm using the notation of a mathematical function: read $e_{sat}(T)$ as "the value of the function e_{sat} at temperature *T.* In any event, if we specify the raindrop temperature we can calculate the water vapor pressure at its surface. I've added here a graph of e_{sat} vs. temperature:



Vapor pressure and relative humidity: The water vapor pressure in the air is commonly referred to indirectly, when one states the *relative* humidity, the fraction of saturated water vapor pressure that the air holds. We have to know the air temperature at the same time, thus, its saturated value. Let us write

$$e_{air} = h_r e_{sat}(T_{air})$$

Here, the relative humidity is specified as a fraction, vs. a percent (recall that percent is per 100); we just multiply the saturated vapor pressure by the fraction.

We end up with this expression for the cooling rate:

$$C = \frac{\lambda g_{bw}}{P}\left[e_{sat}(T_{drop}) - h_r e_{sat}(T_{air})\right]$$

Simplifying approximation for vapor pressure at raindrop temperature: A challenge here is that we may know air temperature but not the raindrop temperature. We have to solve for it, in fact, making it the primary (only) variable. Let's express it in terms that mean the most, its difference from air temperature. Call this difference ΔT, "delta-T", the drop in T:

$$T_{drop} = T_{air} - \Delta$$

We're going to end up with an equation just for Δ.

We need to express $e_{sat}(T_{drop})$ in terms of T_{air} and Δ. For any smooth mathematical function like that of Murray's equation, we can approximate its value at one point (say, T_{drop}) by its value at another (say, T_{air}) and its rate of change, or slope. Call this slope *s*:

$$e_{sat}(T_{drop}) \approx e_{sat}(T_{air}) + s * [T_{drop} - T_{air}]$$
$$\approx e_{sat}(T_{air}) - s\Delta$$

The wiggly equal sign means "approximately equal to." (Using calculus, we can compute *s* as

$$s = e_{sat}(T_{air})\left[\frac{4098}{(237.2 + T_{air})^2}\right]$$

If T_{air} varies only 5 or 10 degrees from, say, 25°C, the slope is rather constant, about 0.06 or 6% per degree.)

Now we're ready to express the cooling rate in terms of just air temperature, relative humidity, air pressure, the (ultimately irrelevant) boundary-layer conductance, and the single unknown, Δ:

$$C = \frac{\lambda g_{bw}}{P}\left[e_{sat}(T_{air})(1-h_r) - s\Delta\right]$$

I really should use an approximately-equal sign, of course.

Now for a simpler part, the heating rate, *H*. This is just the boundary-layer conductance for heat transfer, g_{bH}, multiplied by a driving force proportional to the difference in temperature between the air and the raindrop. The driving force is the temperature difference, Δ, multiplied by the heat capacity of air, which we'll denote as C_P. The subscript *P* indicates it's evaluated at constant air pressure. We're using molar units, so this is a constant, 29 joules per mole per degree (degree Celsius, or Kelvin).

$$H = g_{bH} C_P \Delta$$

That was much simpler than the cooling rate!

Another simplification now is that the two boundary-layer conductances are very nearly equal

$$g_{bw} \approx 1.1 g_{bH}$$

Basically, water molecules are lighter and more mobile than the air molecules that move heat.

Solving for the temperature drop, Δ: We can now equate the cooling and heating rates for the drop that reaches steady state as it falls:

$$\frac{\lambda g_{bw}}{P}\left[e_{sat}(T_{air})(1-h_r) - s\Delta\right] = g_{bH} C_P \Delta$$

Using algebra, let's gather terms in Δ on the one side; at the same time, replace g_{bw} by $1.2 g_{bH}$ and cancel out g_{bH} on both sides

$$\Delta\left[C_P + \frac{1.1\lambda s}{P}\right] = \frac{1.1\lambda s}{P} e_{sat}(T_{air})[1-h_r]$$

$$\rightarrow \quad \Delta = \frac{\frac{1.1\lambda s}{P} e_{sat}(T_{air})[1-h_r]}{C_P + \frac{1.1\lambda s}{P}}$$

Clean it up a bit; multiply top and bottom by *P*:

$$\Delta = \frac{1.1\lambda e_{sat}(T_{air})[1-h_r]}{PC_P + 1.1\lambda s}$$

This has the right behavior. As the humidity decreases, $1\text{-}h_r$ increases – the cooling is greater in dry air that allows more evaporation, more cooling.

We're ready to use it. Let's take the case of 50% relative humidity ($h_r = 0.5$) and an air temperature of 30^{o}C, a warmish, not too humid day. At this temperature, e_{sat} is 4246 Pa and the slope is 244 Pa per degree. Let's be near sea level and put $P = 100{,}000$ Pa. We get

$$\Delta = \frac{1.1 * 44,000 J\,mol^{-1} * 4246 Pa * 0.5}{10^5 Pa * 29 J\,mol^{-1}\,K^{-1} + 1.1 * 44,000 J\,mol^{-1} * 244 Pa\,K^{-1}}$$
$$= \frac{1.03x10^8}{1.47x10^7}$$
$$= 6.99K\ (6.99°C, 12.6°F)$$

I put in multiplication signs explicitly for clarity and I went to exponential notation, 100,000 = 10^5. We see that the cooling is significant! Raindrops at 23°C falling on skin at probably 32°C feel quite cold.

Measuring relative humidity, instead: We can invert the process. Let's go out on the same day and measure both the air temperature and the raindrop temperature. From that we can compute the relative humidity. We'll rearrange the equation for cooling equal to heating:

$$1 - h_r = \frac{\Delta\left[PC_P + 1.1\lambda s\right]}{1.1\lambda e_{sat}(T_{air})}$$
$$= \frac{6.99K[10^5 Pa * 29 J\,mol^{-1}K^{-1} + 1.1 * 44,000 J\,mol^{-1} * 244 Pa\,K^{-1}}{1.1 * 44,000 J\,mol^{-1} * 4246 Pa}$$
$$= \frac{1.03x10^8}{2.06x10^8}$$
$$= 0.5$$

Using it this way is essentially the principle of the sling psychrometer: with one thermometer wetted (reaching the cooled steady state, after about 1 min of fast rotation in the air) and one dry (reaching air temperature), one obtains the value of Δ. There's a scale that varies with air temperature, accounting for the dependence of e_{sat} upon air temperature. One goes to the correct scale and gets relative humidity.