Fixing approximations / shortcuts in big-leaf model

I have some concerns that the approximations in the big-leaf model (mine, or any big-leaf model) might compromise the accuracy of predicting the responses of photosynthesis and water-use efficiency to water and N stresses.

(You can jump to the radiative transport section, where I deal with second all orders of light scattering)

1) The big-leaf model uses a single, average level of leaf irradiance, I_L, in computing rates of photosynthesis and transpiration

This introduces errors:

In contrast to this assumption:
a) my earlier work established that, for sunlit leaves with random leaf angle distributions, there is a uniform probability of any irradiance between zero and full normal incidence (I_N);
b) photosynthetic rate, A, is a convex function of I_L; consequently, the average value over a range of I_L is greater than the rate at the average value of I_L; and
c) leaves at a given location in the canopy have a certain average value of diffuse light, in addition to the direct solar beam. This diffuse light arises from diffuse skylight penetrating from diverse directions, as well as from scattering of the direct beam from other leaves. Other work of mine (Gutschick, 1984) showed that the probability distribution of diffuse light is narrow. One can consider the diffuse light as occurring only at its average value.   For a layered canopy (laterally uniform), this value, $D(L)$, is a function only of depth in the canopy, measured as cumulative leaf area index, L, from the top.
Consequently, we want to replace the average photosynthetic rate in the canopy by the average over irradiances from $D(L)$ to $D(L)+I_N$.

Getting a good average of photosynthetic rate over the range of irradiance:

A very good analytic (closed-form) expression for A as a function of irradiance, $A(I_L)$, is the Johnson-Thornley form. It accounts for the transition between the light-limited rate, A_{LL}, that is linear in irradiance ($A_{LL} = Q_0 I_L$), and the the maximal rate, A_{max}:

$$\theta A^2 - A(A_{max} + A_{LL}) + A_{max} A_{LL} = 0$$

The value of θ, which lies between 0 and 1, indicates the sharpness of the transition; a value near 1 represents a sharp transition. This equation can be solved for A:

$$2\theta A = A_{max} + Q_0 I_L - \sqrt{(A_{max} + Q_0 I_L)^2 - 4\theta A_{max} Q_0 I_L}$$
$$= A_{max} + Q_0 I_L - \sqrt{A_{max}^2 + (2 - 4\theta) A_{max} Q_0 I_L + Q_0^2 I_L^2}$$
$$\equiv A_{max} + Q_0 x - \sqrt{a + bx + cx^2}, \quad x = I_L$$

We want to integrate this from $x_0 = D(L)$ to $x_f = D(L)+I_N$, with uniform weighting, to get the average value,

$$2\theta\bar{A} = \frac{\displaystyle\int_{x0}^{xf} dx\, A(x)}{\displaystyle\int_{x0}^{xf} dx} = \frac{A_{max}I_N + \frac{1}{2}Q_0[(I_N + D(L))^2 - D(L)^2] - \int_{x0}^{xf} dx\sqrt{a+bx+cx^2}}{I_N}$$

$$= A_{max} + \frac{1}{2}Q_0[I_N^2 + 2I_N D(L)] - \frac{1}{I_N}\int_{x0}^{xf} dx\sqrt{a+bx+cx^2}$$

The integral of the radical at the right-hand end is given in standard tables, e.g., Gradshteyn and Rhyzyk:

$$\int dx'\sqrt{R(x')} = \frac{(2cx+b)}{4c} + \frac{\Delta}{8c}\int \frac{dx'}{\sqrt{R(x')}}, \quad where \; \Delta = 4ac - b^2$$

and, finally,

$$\int \frac{dx'}{\sqrt{R(x')}} = \frac{1}{\sqrt{c}}\ln[2c\sqrt{R(x)} + 2(x+b)]$$

We need to evaluate this integral at the upper and lower limits, subtracting the latter from the former.

Consider fixed values of Q_0 (a modest function of temperature) and θ (values near 0.8 are very common among diverse species). We can infer from the analytic form above that there is a simple regularity: $\bar{A}$, normalized by A_{max} (that is, $\bar{A}/A_{max}$) is a function only of the limits as I_L/A_{max}. This can be seen in the graph below, for calculations with a zero lower limit and various values of the upper limit, but with two different values of Amax. The curves lie on top of each other:

theta=0.8, Q0=0.05
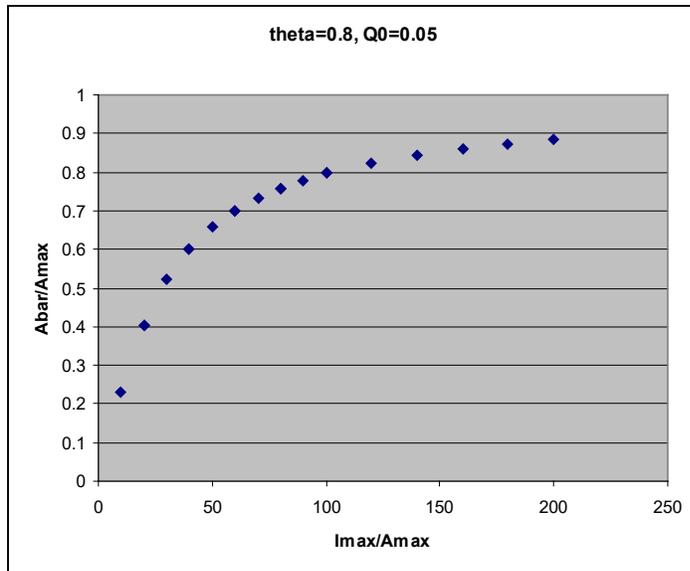
Going back to the original equation for 2θA, we can see that it can be rewritten in terms of the ratio, y = Q0IL/Amax, as $A_{max}[1 + y - \sqrt{1 + (2 - 4\theta)y + y^2}]$.

We can use the analytic form for $\overline{A}$. Alternatively, we can do a numerical integration, using 10 evenly-spaced discrete points between *D(L)* and *D(L)+I*$_N$. This takes only a bit more computation, and the effort is a minor part of the entire computation.

Proper averaging of the photosynthetic rate in this manner can be implemented in any big-leaf model, adding little to the computational effort.

2) I used, as do others, only first interceptions of radiation to represent the radiation load on leaves.

Second and higher-order interceptions are important in both energy balance and driving photosynthesis

However, light scatters from other leaves and from soil, and this represents a significant part of the total radiation (energy) load on leaves. This is particularly true for near infrared radiation (NIR), of which only about 35% is absorbed on first interception but of which twice as much or more is ultimately absorbed. Ignoring these diffuse fluxes gives us an error in leaf temperature, thus also in rates of photosynthesis and transpiration.

We could simply inflate the radiation load of PAR and NIR on the average leaf, to account for the ultimate absorption by the canopy. We're led to look at the details of how the flux density of radiation varies with depth, for two reasons. First, we need to know how the reflectance fraction varies with canopy total depth (leaf area index, LAI) and leaf-angle distribution (LAD, especially considering some major types such as random and horizontal). Second, we want to see how varied the loads are on leaves in the canopy. If the distribution of PAR and total energy loads is complicated, we may want to go to a simple multi-layered canopy model. This will be easier than the elaborate canopy model that tracks radiation to 125 locations in ellipsoidal crowns, and it may suffice (we can compare our results to those for the ellipsoidal crowns, at least for first interceptions).

Representing the mix of radiative fluxes in a canopy

The simplest representation of radiation fluxes in a canopy is resolving the direct solar beam and then diffuse fluxes as downward and upward diffuse fluxes. That is, we don't resolve all the details of the angular distribution of diffuse fluxes. This is a common simplification used in many models such as SAIL. (I have done models that resolve such details; the computational effort is less than elaborate ray-tracing, which is the ultimate model, but it is still very large). The simplification is defensible, based on the more elaborate studies (which show that row structure and random plant-height variation is somewhat more of a modifier of light interception). It remains important to keep the representation of direct light separate from that of diffuse light.

How does diffuse light propagate through a canopy with laterally uniform layers? We will use the notation that $D(x)$ is the downward-propagating flux (as a flux density, to be sure, to keep units clear) and $U(x)$ is the upward-propagating flux, with x as the depth in the canopy as cumulative LAI, measured from the top. That is, $x = 0$ is the top of the canopy and $x = xf = $ total LAI is the bottom of the canopy.

At each incremental layer, from x to $x+dx$, the fluxes are absorbed and scattered. The absorption rate of $D(x)$ is proportional to $D(x)$ and to a diffuse-light absorption coefficient, a:

$$D(x + dx) - D(x) = -aD(x) - sD(x) + sU(x)$$

The last term represents the scattering downward of some of the upward-propagating flux, with a scattering coefficient s, converting this part into downward-propagating flux. The absorption and scattering coefficients for a leaf layer (with some leaf-angle distribution, LAD) are related to the same coefficients for light hitting single leaves (call these a_{leaf}, s_{leaf}). What is the relation? It depends on the angular distribution of the diffuse fluxes and the LAD; a equals a_{leaf} multiplied by the extinction coefficient for a diffuse flux encountering leaves of the specified LAD. For D and U modelled as isotropic fluxes, it turns out that a is closely equal to a_{leaf}, for a variety of different LADs.

There is also forward scattering by leaves, but that simply continues as flux in the same direction, so it does not enter into the equation.

There is a similar equation for the variation of $U(x)$. We can make the same arguments, but take the step upward (going more negative in x), to get

$$U(x) - U(x + dx) = -aU(x) - sU(x) + sD(x)$$

Let's write these as differential equations:

$$\frac{dU}{dx} = -(a + s)D + sU$$

$$\frac{dU}{dx} = +(a + s)U - sD$$

How can we solve the differential equations for the diffuse fluxes?

We can solve these numerically, which is a useful way when a and s vary with depth in the canopy, for example. For constant a and s, we can get an analytical solution….and it may be very close even for the case of varying a and s (as we shall see). An efficient way to solve these equations is to diagonalize them:

Define $A = D + bU$. Then,

$$\frac{dA}{dx} = \frac{dD}{dx} + b\frac{dU}{dx} = -(a+s)D + sU - bsD + b(a+s)U$$
$$= [-(a+s) - bs]D + [s + b(a+s)]U$$

We can make this equation look like a simple first-order d.e. with an exponential solution, if we choose b such that the second bracket is simply b times the first bracket:

$$\frac{dA}{dx} = kD + bkU = kA$$

This requires that

$$k = -[a + s + bs]$$
$$kb = s + ba + bs = -b[a + s + bs]$$

$$\Rightarrow b^2s + 2b(a+s) + s = 0$$

We can solve the last equation for the value of b:

$$b = \frac{-2(a+s) \pm \sqrt{4(a+s)^2 - 4s^2}}{2s}$$
$$= -(1 + \frac{a}{s}) \pm \frac{\sqrt{2as + a^2}}{s}$$

There are two solutions, which is right, because there must be two independent solutions, matching the two original variables D and U. We will call these A_+ and A_-. You can verify that this equation for b has the right limits. For example, as $s \to 0$ (no scattering, only absorption), we get $b = -1$ or infinity. The -1 solution is real and gives the extinction coefficient $k = -[a+s+bs]$ - $-a$, simply. In fact, we can quickly see that the extinction coefficients for the solutions A_+ and A_- are just $-\sqrt{2as + a^2}$ and $+\sqrt{2as + a^2}$, respectively. Call these simply $-K$ and $+K$. (The negative value goes with A_+, whose "+" sign designator is taken from the use of the positive root of the quadratic.)

We get two solutions,

$$A_+ = A_+(0)\exp(-Kx)$$
$$A_- = A_-(0)\exp(+Kx)$$

Now we have to express the solutions for *D(x)* and *U(x)* in terms of $A_+(x)$ and $A_-(x)$.

$$A_+ = D + b_+U$$
$$A_- = D + b_-U$$

We can solve these. Multiply the first equation by b_- and the second by b_+ and subtract:

$$b_-A_+ = b_-D + b_-b_+U$$
$$b_+A_- = b_+D + b_+b_-U$$

$$\overline{}$$

$$b_-A_+ - b_+A_- = (b_- - b_+)D \;\rightarrow\; D = \frac{b_-A_+ - b_+A_-}{b_- - b_+}$$

We can also subtract the first set of equations, to get

$$A_+ - A_- = (b_+ - b_-)U \;\rightarrow\; U = \frac{A_- - A_+}{b_- - b_+}$$

This is where the boundary conditions come in. First, we know the diffuse flux (density) hitting the top of the canopy. Call it D_0. This gives us one equation:

$$D_0 = \frac{b_-A_+(0) - b_+A_-(0)}{b_- - b_+}$$

Second, we know that, at the bottom of the canopy, $x = x_f$, the diffuse flux is reflected by the soil into upward flux:

$$U(x_f) = rD(x_f)$$
$$\rightarrow \frac{A_-(x_f) - A_+(x_f)}{b_- - b_+} = r\frac{b_-A_+(x_f) - b_+A_-(x_f)}{b_- - b_+}$$

Here, *r* is the reflectivity of the soil for the given type of radiation (PAR or NIR). And, of course, we know that the A's are exponentials in x:

$$A_+(x_f) = A_+(0)\exp(-Kx_f)$$
$$A_-(x_f) = A_-(0)\exp(+Kx_f)$$

So, everything is now expressed in terms of known quantities (a and s, which give us b_-, b_+, and K; $D0$; and r), and $A_+(0)$ and $A_-(0)$. We can then solve for $A_+(0)$ and $A_-(0)$, going on to then solve for $D(x)$ and $U(x)$ at all x.

The algebra gets a little tedious. I give the solutions:

$$A_+(0) = \frac{(b_- - b_+)}{[b_- - b_+ \exp(-2Kx_f)\dfrac{(1+rb_-)}{(1+rb_+)}} D_0$$

$$A_-(0) = \frac{(b_- - b_+)\exp(-2Kx_f)\dfrac{(1+rb_-)}{(1+rb_+)}}{b_- - b_+ \exp(-2Kx_f)\dfrac{(1+rb_-)}{(1+rb_+)}} D_0$$

We can use these solutions to get $D(x)$ and $U(x)$ at any point. We can also solve for the reflectance of the canopy for the incident diffuse radiation. Again with some algebra, whose details I skip,

$$U(0)/D_0 = \frac{\exp(-2Kx_f)R - 1}{b_- - b_+ \exp(-2Kx_f)R}$$

Here, R is the ratio appearing earlier, $(1+rb_-)/(1+rb_+)$. For thick canopies, the exponential factors are small, and the canopy reflectivity is close to $-1/b_-$, a quickly calculable quantity. Knowing also that little radiation gets through a thick canopy to hit the soil, we can say to the total amount of radiation absorbed per unit area in the canopy is closely $(1+1/b_-)D_0$. The factor $(1+1/b_-)$ is less than 1, because b_- is negative.

We need to account separately for the direct solar beam, which propagates differently

This treatment is incomplete. We have to account for the direct beam, which has different scattering and absorption coefficients from those for diffuse light. Also, it has a zenith angle, which also affects the extinction coefficient.

In effect, the direct beam, I(x), acts as an independent source, generating diffuse fluxes as it hits leaves. We have to add terms to the original differential equations for D and U:

$$\frac{dD}{dx} = -(a_{diff} + s_{diff})D + s_{diff}U + f_{dir}I$$

$$\frac{dU}{dx} = +(a_{diff} + s_{diff})U - s_{dif}fD + s_{dir}I$$

We are now making a distinction between scattering of the direct and diffuse beams. Earlier, I argued that a_{diff} and s_{diff}, for absorption and scattering of diffuse radiation, are close to the values for single leaves. For the direct beam, we have to consider leaf and beam angles. Consider first the probability of the direct beam encountering leaves along a path ds through a canopy layer, a

path that is at an angle θ_s from the zenith. The formula is well known from canopy theory. It is $<|cos\theta_{Ls}|>ds$. Here, $cos\theta_{Ls}$ is the cosine between the leaves (normal or perpendicular vectors) and the direct beam; it represents the fraction of the flat leaf area that the leaf presents to the beam. The absolute value marker, | |, means that encounters from above or below are close to equivalent (there are small differences in absorption and scattering from the two directions, but we can ignore these). The brackets, $< >$, indicate that we average over the possible angles that leaves take. Two simple cases are illustrative. For leaves that are random in their zenith and azimuthal angles, the factor in brackets is ½. For leaves that are all horizontal, the factor is $cos\theta_s$, the cosine of the solar zenith angle.

Our equations are per unit of vertical distance, dx. The angled path is longer: $ds = dx/cos\theta_s$. We can then say that the encounter probability per unit of vertical path is

$$K_{dir} = \frac{<|\cos\theta_{Ls}|>}{\cos\theta_s}$$

This is a formula that is also familiar from canopy theory. We can now formulate the absorption and scattering coefficients for the direct beam as

$$a_{dir} = a_{leaf} * K_{dir}, \quad s_{dir} = s_{leaf} * K_{dir}, \quad f_{dir} = f_{leaf} * K_{dir}$$

Here, a_{leaf} is the absorption fraction for radiation hitting a leaf (assumed independent of angle of incidence, which is not exactly true), s_{leaf} is the backscattering or reflection fraction, and f_{leaf} is the forward-scattering or transmission fraction. There is another approximation here, about directions of the diffuse light. If the leaves are not horizontal, the angled encounters that generate, say, transmitted flux do not generate purely downward diffuse flux; some of it is actually upward flux. The complications in math to deal with this are rather extreme (Ranga Myneni and I, with two other authors, have dealt with this) and are not worth the increment in accuracy for our purposes.

We can now continue. We can diagonalize the combinations of D and U:

$$\frac{dA}{dx} = -(a+s)D + sU + (f_{dir} + bs_{dir})I$$
$$\equiv kA + (f_{dir} + bs_{dir})I$$

For a little simplicity, I have left off the "diff" subscripts on a and s applied to diffuse fluxes. The very same solutions for b_+ and b_- obtain here as in the original form without the direct beam.

How can we solve these differential equations with the extra "driving" terms?

Now, we know in mathematics how to handle "driving forces" (the term in I here) that are added to a differential equation for what otherwise has an exponential solution. Let's denote $A_0(x)$ as the solution without the driving force - that is, the solutions found earlier, such as $A_{+0}(x) = A_{+0}(0)exp(-Kx)$:

$$A(x) = A_0(x) + (f_{dir} + bs_{dir}) \int_0^x dx' \exp(-K(x-x'))I(x')$$

You can verify that this is correct by taking the derivative, d/dx, of both sides, remembering that the seond term on the right has a derivative of the stuff inside as well as the derivative of the upper limit to the integral. We can now use the simple fact that I(x) falls off exponentially with depth as more and more layers are encountered:

$$I(x) = I(0) \exp(-K_{dir}x)$$

Here, K_{dir} is exactly the extinction coefficient discussed a little earlier, and defined in terms of the sun angle and the leaf angle distribution. Note that *I(x)* is the flux density on a horizontal surface. If the solar beam comes in at an angle off the zenith (not overhead, that is), then the flux density should be measured on a horizontal surface, not perpendicular to the beam itself.

With *I(x)* given by the exponential form, we can, with a little calculus and a little algebra, get some closed-form solutions for the term with the integral:

$$A_+(x) = A_+(0)\exp(-Kx) + (f_{dir} + b_+ s_{dir})I(0)\frac{\exp(-K_{dir}x) - \exp(-Kx)}{(K - K_{dir})}$$

$$A_-(x) = A_-(0)\exp(+Kx) + (f_{dir} + b_- s_{dir})I(0)\frac{\exp(Kx) - \exp(-K_{dir}x)}{(K + K_{dir})}$$

We arrive at the final forms for *D(x)* and *U(x)* again by applying the boundary conditions:. First, at the top of the canopy, the form is unchanged from the case without a direct solar beam:

$$D_0 = \frac{b_- A_+(0) - b_+ A_-(0)}{b_- - b_+}$$

At the bottom of the canopy, the algebraic expression is more complex than in the case without the direct beam:

$$U(x_f) = r[D(x_f) + I(0)\exp(-K_{dir}x_f)] = \frac{A_-(x_f) - A_+(x_f)}{(b_- - b_+)}$$

We can express the middle and right sides all in terms of A_+ and A_-. After some algebraic manipulation, we get the second relation between $A_+(0)$ and $A_-(0)$ as

$$A_-(0) = R\exp(-2Kx_f)A_+(0) + P_2 - P_1 + P_3$$

where *R = (1+rb-)/(1+rb+)*, as earlier, and

$$P_1 = \exp(-Kx_f)(f_{dir} + b_- s_{dir})I(0)E_+$$

$$P_2 = R\exp(-Kx_f)(f_{dir} + b_+ s_{dir})I(0)E_-$$

$$P_3 = \frac{r(b_- - b_+)}{(1 + rb_+)}I(0)\exp(-(K + Kdir)x_f)$$

with the shorthand notation

$$E_- \equiv \frac{\exp(-K_{dir}x) - \exp(-Kx)}{(K - K_{dir})}$$

$$E_+ = \frac{\exp(Kx) - \exp(-K_{dir}x)}{(K + K_{dir})}$$

Substituting the relation between $A_+(0)$ and $A_-(0)$ obtained from the top-of-canopy boundary condition, we finally arrive at

$$A_+(0) = \frac{b_+(P_2 - P_1 + P_3) + (b_- - b_+)D_0}{b_- - b_+ R\exp(-2Kx_f)}$$

We can plug this into the original definitions of $A_+(x)$ and $A_-(x)$ to compute $D(x)$ and $U(x)$ at any x. These equations are, you may recall from earlier,

$$D(x) = \frac{b_- A_+(x) - b_+ A_-(x)}{b_- - b_+}$$

and

$$U(x) = \frac{A_-(x) - A_+(x)}{b_- - b_+}$$

This is a lot of algebra and a bit of calculus, but we end up with a relatively compact set of equations for some complex physics.

Some illustrative results

I have programmed these equations in Fortran (canopy_abs+scatt.f90, available) and also in Excel (trying_scatt+absorp_varied_leaf_prop3.xls, which has advanced features; also available). The programs can compute the fluxes at any depth in the canopy. They can also summarize the disposition of energy:

Inputs: D_0 and $I(0)$
Outputs: Absorption at the soil = $(1-r)(D(x_f)+I(0)exp(-K_{dir}x_f)$
          Absorption in the canopy, by the leaves = Inputs - (Abs. at soil)
          (One can verify the calculation of canopy absorption, obtained in the

line above by difference, by analytically integrating over depths:

$$Abs_{can} = \int_0^{xf} dx[a_{diff}(D(x)+U(x))+a_{dir}I(x)]$$

This is a huge exercise in math and can be simplified by doing the integration numerically. It works.)

Here are some results. In each case, I set the following parameters:
   For PAR: $a_{leaf} = 0.85$, $s_{leaf} = 0.10$, $f_{leaf} = 0.05$
   For NIR: $a_{leaf} = 0.35$, $s_{leaf} = 0.433$, $f_{leaf} = 0.217$
   Extinction coefficient for diffuse flux $= 1$
   Extinction coefficient for direct flux $=$
      $0.5/cos\theta_s$, for random leaves, 1 for horizontal leaves
I used various combinations of incident direct beam ($I(0)$) and diffuse skylight ($D0$).
I used a few different zenith angles for the sun, and two different canopy LAI values, 2 and 4.

Type of radiation	Leaf angles	LAI=xf	D0	I(0)	cosθs	r at soil	Can. relfec. =U(0)/(D0+Abs,can	Abs,soil	Comment
PAR	Random	4	1	0	1	0.1	0.053	0.928	0.020 Yes, rcan<<rleaf
PAR	Random	4	0	1	1	0.1	0.036	0.836	0.130 Less reflec. for all-direct
PAR	Random	4	0.1	0.9	1	0.1	0.037	0.845	0.119 Like normal skies
PAR	Random	4	0.1	0.9	0.7	0.1	0.045	0.902	0.054 Glancing direct beam
PAR	Random	2	0.1	0.9	1	0.1	0.041	0.636	0.325 Thinner canopy
PAR	Horizontal	4	0.1	0.9	1	0.1	0.052	0.927	0.021 Horiz. lvs.
PAR	Horizontal	4	0.1	0.9	0.7	0.1	0.052	0.927	0.135 cosθs has no effect
NIR	Random	4	0.1	0.9	1	0.15	0.222	0.610	0.168 Common case
NIR	Random	2	0.1	0.9	1	0.15	0.213	0.395	0.393 Thinner canopy
NIR	Random	4	0.1	0.9	0.7	0.15	0.262	0.639	0.100 Glancing direct beam

Some features to note include:
- Canopy reflectance is much less than individual-leaf reflectance, esp. for thick canopies. This is as observed in the field.
- Diffuse light is reflected more strongly than direct (overhead) light. Also as observed. The probability of encountering leaves, esp. at the top of the canopy, is higher.
- More glancing incidence of the direct beam leads to enhanced reflectance and enhanced absorption by the canopy, and much less absorption at the soil. This holds for both PAR and NIR
- NIR absorption by the canopy is less than 2/3 that for PAR.
- Thinner canopies have slightly higher reflectance (less deep trapping), considerably less canopy absorption, and much higher absorption at the soil (much higher penetration of the direct beam).

Two sub-topics to pursue, while on the topic of patterns of canopy absorption of radiation:

A) How much different are the depth profiles of absorption for PAR and NIR?
   This is of considerable interest in developing models with more complex canopy geometry, such as the array of ellipsoidal crowns in an orchard in my larger model. It is impractical computationally to model the scattered light going to all other locations (although Sinoquet has a "voxel-by-voxel" method and Chelle and coworkers have a scheme of nested radiosity, which requires knowing where every single leaf is and what its orientation is!). So, in

the orchard model, I assume that PAR and NIR reach any location in the crowns in similar proportions - namely, the same as at the top of the canopy, about 1:1.   How poor is this assumption, which affects the estimation of leaf temperatures and, thus, the estimation of photosynthesis and transpiration rates?

This was easiest to graph in Excel:
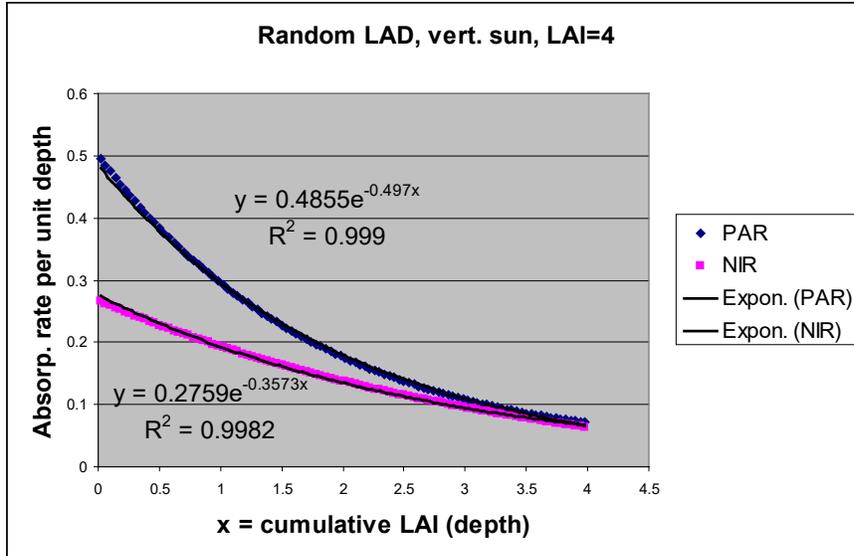
The rates are not fully parallel, but not widely different. We can see this by replotting the rates relative to the top-of-canopy rates:
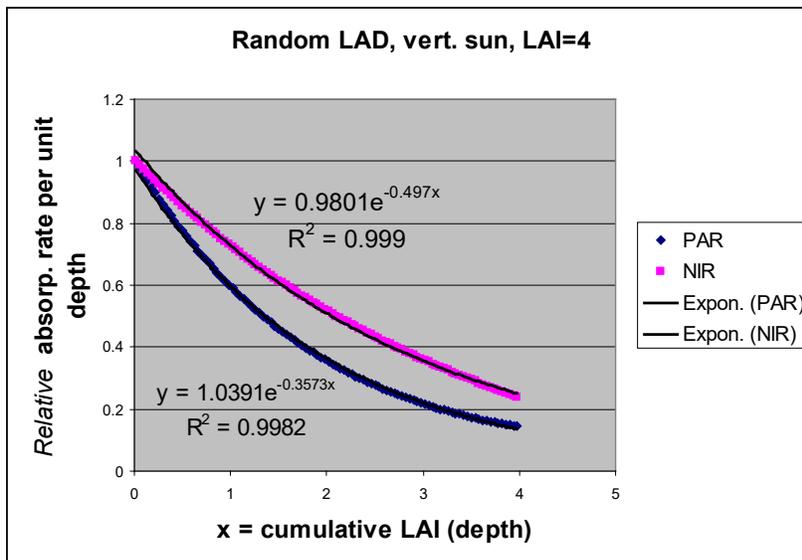
When PAR absorption drops to half its maximal rate, NIR absorption is still about 65% of its maximal rate. Let's put some physically relevant numbers to this. Suppose the condition is midday, and the total energy flux density is 1000 W m^{-2}, approximately half in the PAR and half in the NIR. The maximal absorption rate per unit LAI is 247 W m-2 for the PAR and 138 W m^{-2} for the NIR. At a depth x = 1.34 in leaf area index, the absorption rate for PAR has dropped to

124 W m^{-2} and that for NIR has dropped to 86 W m^{-2}. The simple assumption is that NIR dropped to half its original rate, or 69 W m^{-2}. The error is 17 W m^{-2}, which is not bad. My previous experience with models and measurements indicates that the rate of change of leaf temperature with changes in total energy load is about 1°C per 35 W m^{-2}, so the error in leaf temperature should be about 0.5 °C.

These kinds of errors need not be "imposed" so simply. The error can be completely avoided in layered canopies. In the complex orchard canopy, we can't model the total scattering of either PAR or NIR. However, for random leaves, it appears that the rate of PAR absorption is very closely proportional, at any location, to the rate of first interceptions, which we can model. We can assume the same for NIR, but this would give an underestimate of total NIR absorption, which falls off more slowly. In this case, we can rescale the maximal NIR absorption so that the canopy-integrated absorption is correct.

B) In real canopies, leaves get lighter in color and thinner with depth. The leaf absorption coefficient, a_{leaf}, declines. Total scattering increases, since the sum of absorption and scattering coefficients is unity. Forward scattering becomes closer to backscattering, vs. backscattering being twice the forward scattering for sun leaves. How much difference does this make in canopy absorption, in total and in its pattern with depth?

There is no analytical solution to this case, so I solved it numerically in the Excel program. With a numerical solution, there is no "guidance" for calculating $U(0)$, which basically sets all the other variables at all depths. Handily, we can exploit the linearity of the differential equations: any combination of solutions is a solution. I took $U(0) = 0$ and $U(0) = 1$ and got their results. The important thing is to match the boundary condition at the bottom of the canopy,

$$U(x_f) = r[D(x_f) + I(0)\exp(-K_{dir}x_f)]$$

The two solutions each give a value for $U(x_f)$ and $D(x_f)$ (and the same value for the direct beam). For $U(0) = 1$, we will call the results U_{f1} and D_{f1}. For $U(0) = 0$, we will call the results U_{f0} and D_{f0}. For any change in $U(0)$ to the value 1 to $U(0) = 1 + dU_0$, we have the linear change

$$U(x_f) = U_{f1} + \frac{d(U(x_f))}{d(U(0)} dU_0$$

and similarly for $D(x_f)$. We can then match the boundary condition:

$$U_{f1} + \frac{d(U(x_f))}{d(U(0))} dU_0 = r[D_{f1} + \frac{d(D(x_f))}{d(U(0))} dU_0 + I(x_f)$$

This can be solved for dU_0 by collecting terms:

$$dU_0 = \frac{U_{f1} - r[D_{f1} + I(x_f)]}{\left[r\frac{d(D(x_f))}{d(U(0))} - \frac{d(U(x_f))}{d(U(0))} \right]}$$

We get the derivatives numerically from the two original solutions, as simple differences, such as

$$\frac{d(U(xf))}{d(U(0))} = \frac{U_{f1} - U_{f0}}{1 - 0} = U_{f1} - U_{f0}$$

We rerun the calculation with the new value of $U(0)$ and collect the results.

Using previous models and data (Gutschick, 1984b,c), I set parameters for the bottom leaves as:
  For the PAR: a_{leaf} = 0.75 (from 0.85), s_{leaf} = 0.125 (from 0.1), f_{leaf} = 0.125 (from 0.05).
  For the NIR: a_{leaf} = 0.3 (from 0.35), s_{leaf} = 0.3 (from 0.433), f_{leaf} = 0.4 (from 0.217).
I used a single case: random leaf angles, LAI = 4, vertical sun, 10% diffuse skylight.
The changes in energy distribution were very minor:

	Canopy reflec.	Canopy abs.	Soil abs.
For the PAR, constant parameters:	0.037	0.845	0.119
varying parameters:	0.041	0.833	0.127
For the NIR, constant parameters:	0.222	0.610	0.168
varying parameters:	0.214	0.587	0.201

The conclusion is, Don't worry about this variation.

3) In real canopies, leaves are clumped.

They intercept light less efficiently than leaves whose centers are placed randomly, as was assumed in the equation for the extinction coefficients, Kdiff and Kdir. This changes the fraction of leaves that are sunlit vs. shaded, and it changes the mean values of diffuse fluxes on leaves. How can we account for this? I assume that we can estimate the clumping factor in the field, such as by measuring the real leaf area (by harvesting leaves) and comparing the direct-beam extinction with the predictions from "random medium" theory used so far.
The effect of clumping on flux propagation is to change the extinction coefficients by a factor we may call C: $K_{diff} \rightarrow CK_{diff,r}$; $K_{dir} \rightarrow CK_{dir,r}$. The additional subscript r denotes the estimates for randomly-placed leaves. I apply the same factor C to diffuse and direct fluxes, while it may differ slightly for the two cases. Generally, C is less than 1 (positively clumped leaves). In some canopies, it is greater than 1 (anticlumped or overdisperse leaves).
Canopy models (full multilayer, or expanded big-leaf models) can (should) calculate separately the rates of photosynthesis and transpiration by sunlit leaves and shadelit leaves. Let's first consider the case of unclumped leaves. As mentioned earlier, for randomly-angled leaves, the direct beam hits leaves at depth x with a probability $f_{sun} = exp(-K_{dir}x)$. On that fraction of leaves, there is a range of irradiances from 0 to I_N, the latter being full-on (normal-incidence) sunlight. The average value is, of course, half of this. So, the sunlit leaves in a layer spanning x to x+dx intercept an amount of light $0.5*I_N*fsun*dx$. This correctly accounts for energy disposition. Recall that our differential equation could be used to calculate this same energy absorption,

$$I(x) - I(x + dx) = I(0)[\exp(-K_{dir}x) - \exp(-K_{dir}(x + dx))]$$
$$= I(0)\exp(-K_{dir}x)K_{dir}dx$$
$$= I(0)f_{sun}K_{dir}dx$$

Now, for randomly-angled leaves, $K_{dir} = 0.5/cos\theta_s$, and $I(0) = I_N cos\theta_s$, so that we can rewrite the last line as

$$I_N\cos\theta_s f_{sun}(0.5/\cos\theta_s)dx = 0.5 I_N f_{sun}dx$$

The formulas agree.

Back to work: we simply introduce the clumping factor C into the extinction coefficients, thus also into the absorption and scattering coefficients for leaf layers. Is this equivalent to a simple rescaling, that is, does a canopy with, say, $LAI = 6$ and $C = 2/3$ have the same behavior as one with $LAI = 4$ and $C = 1$? Not exactly. The penetration of the direct beam to the soil level is the same, and the total sunlit area is the same, but the total shadelit area is larger. Consequently, the average diffuse-flux absorption on shadelit (and sunlit) leaves is decreased, by a factor that we can readily compute.

In summary, the effect of clumping is potentially huge, or potentially small, depending on how we describe our canopy. If we use an effective $LAI = C*(true LAI)$, we get the sunlit leaf fraction accurately, but the average irradiance on these leaves is lowered a bit, by the lowering of the diffuse flux absorption. The shadeleaf irradiance is lower but the fraction is larger. Given that the scalings are nearly inverse to each other, and photosynthesis is linear in irradiance at low light levels, we might get shade-leaf photosynthesis fairly accurately. The one caveat is that the radiation load on the average leaf is overestimated, so also is the leaf temperature. The effect is rather modest. I have not taken the (considerable) effort to quantify it.

I have done one estimation for sunlit leaves. When we account only for the direct-beam interception, in a canopy with random leaf angles we expect the rate of absorption of all PAR by the sunlit leaf fraction to be $K_{dir}a_{leaf}I(0)dx$ in a layer x to $x+dx$. The rate of absorption per unit layer thickness is this value divided by dx. Using the simulations for direct-beam only and a vertical sun, we expect the rate per unit layer thickness to be $0.5*0.85 = 0.425$. With scattered, diffuse fluxes present, we get a higher value. At the top of the canopy, where all leaves are sunlit, we find the computed rate is close to 0.50, about 18% higher. Consider the case of clumping where the diffuse fluxes are spread out over 50% more area and are consequently 2/3 as large, on average. We would more correctly estimate the diffuse flux contribution to be only 12% of the direct-beam contribution, or 6% lower. In full sun, the quantum flux density might be 2000 µmol m^{-2} s^{-1} for normal incidence and half of that, 1000 µmol m^{-2} s^{-1}, on the average sunlit leaf. A 6% error here is 60 µmol m^{-2} s^{-1}. Instead of 180 µmol m-2 s-1 as the lowest total irradiance on the leaf, we should get 120 µmol m^{-2} s^{-1}. How much does this change the average photosynthetic rate (before we account for lower value of true leaf temperature)? We can go back to the averaging of A(IL) in grand section (1) in this document. Let's take Amax = 20 µmol m^{-2} s^{-1}. The average photosynthetic rate, $\overline{A}$, for the real range, 120 to 2120 µmol m^{-2} s^{-1} is Amax*[(average A from 0 to 2120)*2120/2000 - (average rate from 0 to 120)*120/2000] = 20 µmol m^{-2} s^{-1}*[0.846] = 16.92 µmol m^{-2} s^{-1}. For the overestimated range, is is 20 µmol m^{-2} s^{-1}*[0.810*2180/2000-0.200*180/2000]= 0.865*Amax = 17.30 µmol m^{-2} s^{-1}. The relative error is

2.25%. Given that photosynthesis will be lowered, in general, by the modestly lowered leaf temperatures (lower radiative loading), the errors partly cancel. They might even be ignored.

4) Real canopies humidify and cool (or sometimes heat) their own environment.

Leaf fluxes are changed

Transpiration by the various leaf layers humidifies the canopy, as is evident to anyone who has walked in a crop stand. Higher humidity has two contrasting effects on transpiration itself. It decreases the vapor-pressure deficit, VPD, thereby suppressing transpiration. It also induces greater stomatal opening, increasing transpiration. Similarly, leaf transpiration cools the air in the canopy, often below the air temperature in free air above the canopy. This, too, reduces VPD and depresses transpiration. It also may reduce photosynthesis rates, inducing a decline in stomatal conductance. On the other hand, it increases relative humidity at the leaf surface, which increases stomatal conductance and therefore transpiration also. The balance of the effects is not readily predicted; there are no simple models. We need to use coupled process models   - stomatal control, photosynthesis, energy balance, and transport relations - in order to compute these changes in the so-called scalars in the canopy - air temperature, T_{air}; water vapor partial pressure, e_{air}; and even CO_2 partial pressure, C_a.

Models of photosynthesis and transpiration used within larger climate models or ecosystem models make a first correction. They include a resistance for transfer of scalars into or out of the canopy. They then apply the new conditions of T_{air}, e_{air}, and C_a to all leaves in the canopy. My big orchard model does the same. These models may iterate the canopy conditions to get final values of the scalars. Iteration is necessary because the responses of A and E to the scalars are nonlinear.   That is, photosynthesis is not a linear function of, say, eair, varied over a range that centers on the initial estimate that eair is what is found well above the canopy in what we may term free air.

The fluxes respond nonlinearly to the changes in local environment

I simulated a single-leaf response to variations in eair, using all the relevant processes: the transport equations for water vapor and CO_2; the Ball-Berry equation for stomatal control; the Farquhar-von Caemmerer-Berry model of photosynthetic enzyme kinetics; and leaf energy balance. The full set of conditions (air temperature, humidity, windspeed, etc.) and leaf physiology (photosynthetic capacity, PAR absorptivity, Ball-Berry parameters, etc.) is recorded but not of prime interest here. The graph below shows that, at lower values of eair in particular, one cannot reliably predict A from the local slope vs. e_{air}, at least, not for significant deviations, which *do* occur in dense canopies. Note that the tangent was drawn at the baseline value of e_{air}.

Sensitivity of photosynthetic rate to e_{air}
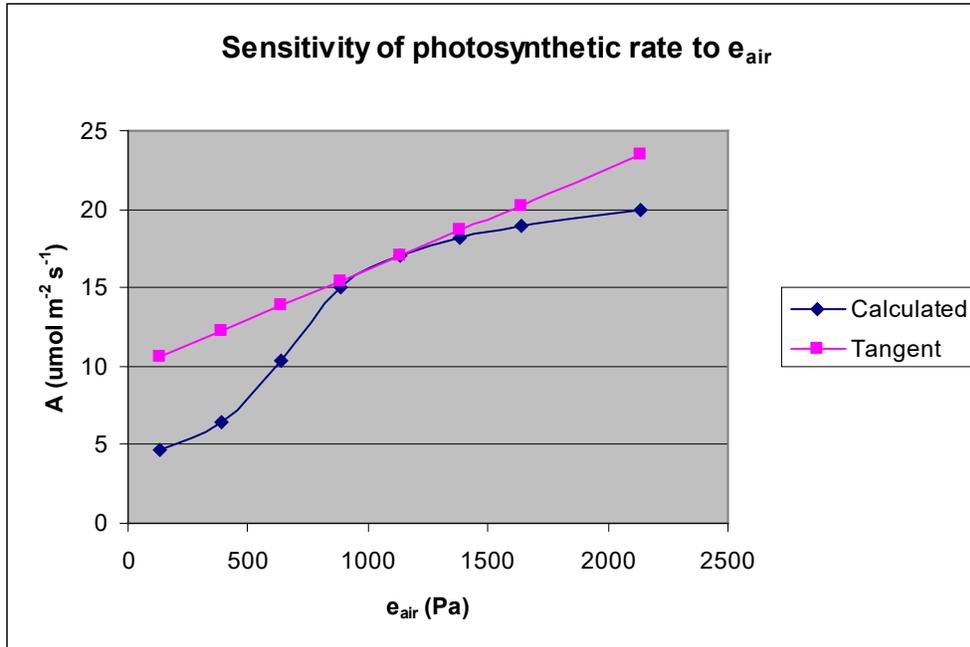
The problem is challenging enough with the assumption that all leaves see the same atmospheric environment: the responses must be formulated properly, and the solutions must be iterated despite tendencies to instability (overshooting the corrections). I have solved this problem in my orchard model, and so have others in their own models that resolve the canopy as a single layer (they may call them two-layer models, the other layer being the soil).

How do the conditions in the canopy vary with depth?

An additional challenge is estimating the effect of diversity of conditions at different locations in the canopy (depths, in the case of a laterally uniform, layered canopy). What happens in one layer, as water vapor injection, sensible heat extraction or injection, and CO2 depletion by photosynthesis, propagates to the adjoining layers. We can model this using an eddy diffusivity. In so-called K-theory, the local diffusivity $d(x)$ is estimated from the top-of-canopy windspeed and the canopy structure. Let us take this as specified, commonly as a decaying exponential such as $d_0 \, exp(-\zeta x)$. Note that K-theory has serious deficiencies when it is used in the inverse mode, to take observed fluxes and infer the strength of sources in each layer. Far superior is Lagrangian transport theory. However, a friend, Chris Bell, and I conferred with a world expert on transport theory, Mike Raupach. He loaned us a Lagrangian transport code. We found that its predictions of fluxes from modelled sources was quite good. Mike later published a paper that noted this satisfactory state of affairs.

The canopy, as a set of layers, can be treated as having sources of (ultimately) known strength or intensity, S(x), at each depth. For water vapor, we may specify this as volume sources, in, say, mol m^{-3} s^{-1}, and it equals the average transpiration rate per unit leaf area, multiplied by the foliage density in this layer. We will denote our scalar (water vapor, or whatever) as having a local concentration, c(x), and a flux J(x) (positive is in the downward direction). The diffusion equation is, as usual,

$$\frac{\partial c}{\partial t} = -\nabla \cdot \vec{J} + S, \qquad \vec{J} = -d\nabla c$$

We're working in only one dimension, the vertical, so we can drop the vector notation, and make all derivatives with respect to x. Moreover, we are interested in steady-state fluxes, so that we can set $\partial c/\partial t = 0$, to get

$$\frac{d}{dx}\left(d(x)\frac{dc}{dx}\right) = -S$$

We can formally divide up the canopy into a reasonable number of discrete layers, indexed by subscript i. The x-coordinate in each layer will be denoted as x_i. We will then set up equations for the fluxes between layers, in terms of the concentration in each layer, c_i. The flux from layer i to layer i+1 is then

$$J_{i,i}+1 = -g_{i,i+1}(c_i - c_{i+1})$$

The conductance between layers is 1 over the diffusive resistance between the layers,

$$g_{i,i+1} = 1/\int_{xi}^{xi+1} dx/d(x)$$

For short distances relative to the scale over which $d(x)$ changes, this is $d(x)/(x_{i+1}\text{-}x_i)$.

The rate at which the concentration builds up in layer i is the volume source strength, $S(x)\Delta x_i$, plus the net flux into the layer, into minus out, $J_{i-1,i} - J_{i,i+1}$. Here, Δx_i is the width of the individual layer. (For unevenly spaced layers, it does not necessarily equal the distance from the center of the layer to either of the neighboring layers.) For steady state, we have to require that this rate is zero in each layer. We get an equation for each layer:

$$S_i\Delta x_i + g_{i-1,i}(c_{i-1} - c_i) - g_{i,i+1}(c_i - c_{i+1}) = 0$$

Setting up and solving a set of linear equations

The whole set of equations is just a set of linear algebraic equations in the set $\{c_i\}$. We can make them even simpler, knowing that the flux in each layer is the sum of the fluxes below plus $S(x)\Delta x_i$ for the layer. As a simple example that we can easily generalize, consider a canopy with three layers, with a free-air layer above having concentration c_0, and a soil layer beneath. We'll set the soil evaporation rate to zero; it is easily incorporated later. We have:

$$\textit{Flux from top layer to free air} = g_{01}(c_1 - c_0) = S_1\Delta x_1 + S_2\Delta x_2 + S_3\Delta x_3$$
$$\textit{Flux from layer 2 to layer 1} = \quad g_{12}(c_2 - c_1) = S_2\Delta x_2 + S_3\Delta x_3$$
$$\textit{Flux from layer 3 to layer 2} = \quad g_{23}(c_3 - c_1) = S_3\Delta x_3$$

This can be set up in linear-algebraic notation as

$$
\begin{pmatrix}
g_{01} & & \\
-g_{12} & g_{12} & \\
& -g_{23} & g_{23}
\end{pmatrix}
\begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}
=
\begin{pmatrix}
g_{01}c_0 + S_1\Delta x_1 + S_2\Delta x_2 + S_3\Delta x_3 \\
S_2\Delta x_2 + S_3\Delta x_3 \\
S_3\Delta x_3
\end{pmatrix}
$$

We (must) know the free-air condition, that is, c_0, and we have computed from the physiology the source strengths in each of the 3 layers. From K-theory we have the values of the $g_{i,i+1}$. We can then invert the matrix on the left and multiply the column vector on the right by this inverse to get the vector of concentrations.

This method is overkill for the simple case. We could simply solve the individual equations from the top down. The linear algebraic equations have been set up to handle the responses of the sources to the concentrations (e.g., the response of transpiration to e_{air} in each layer).

We can write the source strength, with its response to the local ci value as

$$
S_i = S_i^{\,0} + t_i c_i
$$

It may be more useful to write the response relative to a base value, $(c_i - c_i^{\,0})$, but the math is simple to generalize. We can put this into the linear equations to get:

$$
\begin{pmatrix}
g_{01} - t_1\Delta x_1 & & \\
-g_{12} & g_{12} - t_2\Delta x_2 & \\
& -g_{23} & g_{23} - t_3\Delta x_3
\end{pmatrix}
\begin{pmatrix}
c_1 \\
c_2 \\
c_3
\end{pmatrix}
=
\begin{pmatrix}
g_{01}c_0 + S_1^{\,0}\Delta x_1 + S_2^{\,0}\Delta x_2 + S_3^{\,0}\Delta x_3 \\
S_2^{\,0}\Delta x_2 + S_3^{\,0}\Delta x_3 \\
S_3^{\,0}\Delta x_3
\end{pmatrix}
$$

The sensitivities, $\{t_i\}$, have to be computed numerically, by solving all the coupled equations for leaves in layer i with the base set of conditions, then with carefully chosen (relatively small) offsets in T_{air}, e_{air}, and C_a to get the ti as first derivatives.

Linearity is dubious, as seen above. It is best to reduce the sensitivities, $\{t_i\}$, to some fraction of their true values and then iterate. The solution will converge, more slowly but almost always more surely.

The equations for all three scalars at once - T_{air}, e_{air}, and C_a - generate a matrix that is of a dimension three times the number of layers. Regrettably, there are far-off-diagonal elements, such as e_{air} in layer 1 affecting photosynthesis in layer 1, hence affecting C_a in layer 1, which is 10 columns off the diagonal for a 10-layer model. One can still use a sparse-matrix solver to keep decent speed in numerical solutions.

I note that there is a more sophisticated way to solve the equations for scalar transport. Instead of having simple discrete layers, one can retain a continuous view, representing the scalars as a sum (series) of Chebyshev polynomials in depth x). This is the collocation method for solving the differential equations. It's implemented in the Numerical Algorithms Group

Fortran library. I have used it in the past in a multilayer model, though that model is rather old and lacks good accounting for some processes. It is easier to start from scratch.

When conditions change in each layer, so do the exchanges of thermal radiation between layers

The energy balance of leaves in a given layer depend on the local atmospheric conditions (e_{air} and T_{air}) as well as on the radiative fluxes reaching this layer. The propagation of shortwave radiation, the PAR and NIR fluxes, has been solved for by the methods above, at least to acceptable accuracy. However, the thermal infrared radiation (TIR) reaching a layer comes not only from the sky but from other leaves in the canopy. A new phenomenon in TIR, relative to PAR and NIR, is that leaves themselves emit TIR.

We need to account for the propagation of TIR between all the layers. TIR is another diffuse flux, so we can use differential equations to describe its propagation, with the addition of terms that describe the emission by leaves in each layer. Each layer absorbs downwelling and upwelling diffuse TIR flux, and scatters it (so far, same as for shortwave fluxes). The absorption coefficient for leaves is simply the thermal emissivity of leaves, $a_{diff,TIR} = \varepsilon$, with ε close to 1 (about 0.96). The leaves scatter TIR, almost solely backward ($s_{leaf,TIR} \approx 1 - \varepsilon$ and $f_{leaf,TIR} \approx 0$). The layer also emits TIR, in both directions, with an intensity equal to the emissivity multiplied by the blackbody flux density, Q_{TIR}, at its absolute (Kelvin) temperature, $\varepsilon\sigma T_{L,abs}{}^4$. Finally, I note that TIR at the top of the canopy is essentially only from the diffuse sky radiation, emitted by the water molecules in the atmosphere at an effective radiative temperature T_{sky}, which is sometimes well below local air temperature. Sunlight has essentially no TIR, so that there is no direct-beam term.

Modified equations for the propagation of diffuse TIR flux density

The new differential equations for TIR are then

$$\frac{dD}{dx} = -(a+s)D + sU + aQ_{TIR}(x)$$

$$\frac{dU}{dx} = +(a+s)U - sD - aQ_{TIR}(x)$$

Here, a is $\varepsilon K_{diff} \approx \varepsilon$, and s is $s_{leaf}K_{diff} \approx 1 - \varepsilon$.

The equations are solved in a fashion almost exactly analogous to those for the shortwave diffuse fluxes. The numerical solutions are particularly simple, and are appropriate to use when leaf temperature varies with depth in the canopy, as is the universal rule. The only difference from the case of shortwave diffuse fluxes is in the lower boundary condition. Here, the upward flux equals the TIR reflectivity ($1 - \varepsilon$) multiplied by the downward flow, $D(x_f)$, plus the emission from the soil, at a flux density $\varepsilon_{soil}\sigma T_{soil,abs}{}^4$:

$$U(x_f) = (1 - \varepsilon_{soil})D(x_f) + \varepsilon_{soil}\sigma T_{soil,abs}{}^4$$

We can use the same trick of computing solutions with two values of *U(0)* and then combining them to find the exact solution that satisfies the above boundary condition. It is best to use *U(0)* = 0 and *U(0)* = a number on the order of QTIR. While *U(0)* = 1 will work in theory, the major fluxes will have magnitudes on the order of hundreds or thousands (of W m^{-2}), and the small change to *U(0) = 1* will get partly lost in the rounding errors of the larger flux numbers. In any event, once we have the two solutions, we can compute the increment dU_0, measured from the starting point of *U(0) = 0*:

$$dU_0 = \frac{U_{f0} - (1 - \varepsilon_{soil})D_{f0} - \varepsilon_{soil}Q_{TIR,soil}}{(1 - \varepsilon_{soil})\dfrac{d(D_{f0})}{d(U(0))} - \dfrac{d((U_{f0})}{d(U(0))}}$$

I programmed this in Excel, including the accounting for energy disposition. The case with zero incident flux reveals the emissivity of the canopy as the whole. One sets a uniform leaf (and soil) temperature and obtains an outgoing flux, U(0). This divided by the blackbody flux at the chosen temperature, $\sigma T_{leaf,abs}^4$, yield the emissivity. For leaf emissivity of 0.96, the emissivity of a deep canopy (LAI = 4) is 0.98. This agrees with experimental observations. In this case, one also sees a strong radiative deficit for leaves at the top of the canopy, which emit considerably more TIR than they intercept from the lower leaves only. Putting in a downwelling flux from the sky, one sees reasonable TIR deficits. Of course, with the shortwave fluxes, the radiation balance is highest, not lowest, at the top of the canopy. The Excel spreadsheet is named trying_scatt+absorp_varied_leaf_prop3.xls. (It is designed to handle PAR, NIR, and TIR, but I introduced a modest error in the PAR and NIR cases, which will be fixed later.)

The calculations of TIR transport into and within the canopy can be used to improve the accuracy of calculating canopy photosynthesis and transpiration. The TIR calculation can be redone at each iteration of the scalar conditions (*T_air*, *e_air*, *C_a*). In principle, one could define transfer coefficients from each layer to every other layer and handle the exchanges in much the same manner as the transport of scalars. However, the TIR transfer coefficients form a full matrix, not a tridiagonal one, so that they add unnecessary numerical effort.

5) Conclusions and plans

We have just covered several processes that may cause important differences from big-leaf models and even many of the multilayer models. In some cases, I was able to argue, from quantitative results, that the processes generate only minor differences in canopy fluxes, that is, photosynthesis and transpiration. In other cases, it is not yet clear. Consequently, we are not sure of the magnitude of errors resulting from model simplifications. We are also not sure of the magnitude of errors in the flux responses to water and N stresses, which are our goals. We need to run progressively improved models, each with and without the stresses, and see if the models of varying sophistication predict *relative* changes in canopy *A* and *E* under stress that differ among the various models

A number of sequences of model improvements are possible, starting from the big-leaf model as it stands. One possible sequence is:

- Add the extra radiation load from absorption of scattered PAR and NIR, while keeping a single average condition for all leaves in the canopy.
- Do a proper average of A (and E) over the diverse light levels experienced by sunlit leaves (and add the contribution of shaded leaves).
- Compare the full distribution of leaf irradiances for a layered canopy with those for my orchard model that has only first interceptions.
- Develop a true multilayer model, with varied irradiances and temperatures in each layer (and various leaf orientations).
- (Account for leaf clumping; this is less critical, in view of the arguments given in the section on clumping)
- Account for scalar transport and TIR transport in the canopy