Mental math while biking: How big is that cloud, and how high is it?

This is a fun exercise that you can try; it takes some practice.

I was riding my bike on a day with scattered clouds moving across the sky. After several clouds had passed over me, essentially moving directly opposite my track, I realized that I had a quick measure of their size. Of course, I couldn't see their entire shadow on the flattish landscape, but I could see how fast they were moving and how long it took their shadow to pass. To get their speed, or, let's say velocity, since I also knew the direction, I looked at how long the shadow took to pass between two markers whose separation I could estimate easily. I counted the seconds, then measured the distance from the revolutions of my bike pedals, which I knew drove me 4 meters each revolution in the gear I was in. Then I simply divided the distance by the time, to get v = d/t. There are a few mental tricks to long division without a calculator, mostly using successive approximation. I don't recall the exact numbers, but let's say it took 17 seconds for the shadow to cover 100 meters. Then the velocity was about 6 meters per second opposite my direction. I could fine-tune this to 5.9 m/s, knowing that 6*17 is 102, not 100, or 2% high.

Of course, it's hard to keep two counts for time in one's head, so I could only estimate the size of the next cloud passing over, assuming the clouds are all moving together. I stopped riding when the next cloud came over, to get its own velocity, not the algebraic sum of its velocity and mine. Let's say it took 85 seconds. I multiplied this by 6 to get a lateral length of 510 meters – make that 500, subtracting the 2% overestimate of velocity.

Now, how high is it? Clouds can form at many heights above the terrain, and their shape is not a good indicator. What is useful information is how much of an angle a cloud subtends in one's view. Estimates are a bit iffy, of course; we don't usually train ourselves to measure angles by eye. Having done a lot of photography with lenses of different focal lengths gave me an edge. A "standard lens" for a true 35 mm camera has a focal length of 50 mm and a field of view that's about 40 degrees, or 20 degrees to each side of the center. So, I have a feeling for, especially, an angle of 30 degrees, for which the sine is 0.5, meaning that an object subtending 30 degrees in my view is half as wide as it is high. The cloud I viewed looked like it was about ¼ that width – I mentally laid out about 4 copies of it to reach the 30 degrees with which I was familiar. That made the sine of its angle close to ¼ of the sine of 30 degrees (the sine function is pretty linear for small angles). That made the cloud's height above terrain 4 times 2 = 8 times higher than its width, or about 4,000 meters. That's a respectable height for the base of a cloud. Of course, there are corrections if the cloud is viewed significantly off vertical.

The calculations gave me some good mental exercise and a little more insight into what was going on in nature around me.