People will drive some distance to get cheaper fuel (gasoline, diesel) for their vehicle. Of course, they're using fuel to drive that extra distance. How much are they really saving, or, What is the maximal distance for which it's at least as cheap as buying the more expensive fuel?

Assume that the extra travel to get cheaper fuel is a round trip (vs., say, a diversion on some "useful" errand). The cheaper fuel is at a distance *d* in miles from the more expensive station, incurring a round trip with a total distance of *2d*.

Take the price per gallon at the more expensive station as P_1 and the price at the cheaper station as P_2, or $P_1 - \Delta$, with Δ clearly being the difference in price.

Assume that the driver will fill the tank with *g* gallons of fuel. (Immediate realization: a larger fill-up dilutes the added cost of the extra drive.) Further assume that the driver will fill up with *g* gallons, no matter which station he or she goes to; that is, the driver will not buy more fuel at the cheaper, more distant station because the vehicle used up more fuel – the driver plans well and always fills up at a certain level of fuel remaining.

Assume that the vehicle gets *e* miles per gallon, and this does not differ between the usual driving and the drive to the cheaper station (if the cheaper station is in congested traffic, e.g., then it becomes more wasteful of fuel and the tradeoff gets worse.)

We want to compute the "useful distance" traveled, *U*, which is just the total distance one can travel with *g* gallons at *e* miles per gallon or mgp, minus the extra distance, *2d*.

Going to the more expensive station, U = ge
Going to the cheaper station, U = ge – 2d

Let's now compute the useful distance traveled per dollar expended. First, we need the cost, C:

Going to the more expensive station, $C = P_1 g$
Going to the cheaper station, $C = (P_1 - \Delta)g$

We're ready to compare the useful distance per dollar, V (for "value"):

Going to the more expensive station, $V = \frac{ge}{P_1 g} = \frac{e}{P_1}$

Going to the cheaper station, $V = \frac{ge - 2d}{(P_1 - \Delta)g}$

The increase in value going to the cheaper station is then

$$S = \frac{ge - 2d}{(P_1 - \Delta)g} - \frac{e}{P_1}$$

When $S = 0$, there's no benefit (in immediate cost savings) to go to the more distant, cheaper station, and going further is an actual loss. Let's solve for the distance d that is the breakeven point. We'll take the equation above and multiply by $(P_1 - \Delta)g$ to make the algebra easier:

$$ge - 2d = \frac{e(P_1 - \Delta)g}{P_1}$$

This is readily rearranged to get d on one side:

$$2d = ge - \frac{ge(P_1 - \Delta)}{P_1} = ge\left[1 - \frac{P_1 - \Delta}{P_1}\right] = ge\left[\frac{P_1 - P_1 + \Delta}{P_1}\right]$$
$$= ge\frac{\Delta}{P_1}$$

This is a really simple result, and it makes sense piece-by-piece: a greater distance to the cheaper station is supportable if:

- g is greater; the driver buys more fuel, so that the extra distance is diluted into a larger useful distance
- e is greater; the vehicle is more fuel-efficient
- Δ is greater; the price savings per gallon is greater
- P_1 is smaller; the relative price saving per gallon, Δ/P_1, is greater

We've neglected some other considerations that make the tradeoff vanish ($S \rightarrow 0$) at smaller distances:

- Is the driver's time worth something? There's a waste of time going to the cheaper station.
- Fuel isn't the only cost of operating the vehicle. The extra drive puts wear and tear on the vehicle, adding to maintenance costs and depreciating the vehicle (reducing its resale value).
- The federal regulations for deducting mileage costs is 55 cents per mile.
- The US fleet average fuel efficiency is (nominally) 26.4 mpg for cars, light trucks, and SUVs (http://en.wikipedia.org/wiki/Fuel_economy_in_automobiles).
- The US average cost of gasoline varies with some rapidity, but let's take it as \$3.60 per gallon (diesel is higher).
- Thus, the fuel cost per mile is estimated at \$3.60/gallon divided by 26.4 mpg, or 13.6 cents per mile. This is slightly less than ¼ of the estimated total cost of operating a vehicle. So, let's cut the maximal distance by a factor of 4!

$$d = \frac{ge\Delta}{8P_1}$$

Example: taking g as 15 gallons, e as the fleet-average 26.4 mpg, Δ as 10 cents per gallon (\$0.1), and P_1 as \$3.60 per gallon:

$$d = \frac{(15\,gal)(26.4\,mi/gal)(\$0.1/gal)}{8 * \$3.6/gal} = 1.4\,miles$$

That's not very far!...and we haven't counted the waste of time.

Most people don't take the maintenance and depreciation costs into consideration when they make an immediate purchase of fuel. They'll drive a lot farther, up to the 4 times greater distance, and sometimes more. This is at their economic peril in the long term… and we seem to be pretty good at imperiling ourselves financially, considering only up-front costs for everything from fuel to national budgets. Oh, well.