**Calculations – how much heat is saved by turning the thermostat lower at night in our home?**

A foray into energy conservation and a goodly amount of basic physics for scientifically-minded readers

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We have an automated thermostat that we set to attain 70°F after turning on at 6:40 AM and then turn off to let the inside temperature drop as low as 62°F overnight. It could be turned down lower at night, saving some heating. Of course, this leads to a longer stretch of heating to warm up the house in the morning. How much heat is saved? Ultimately, the saving results from having a lower temperature difference between the interior and the outside environment, to which difference the heat loss from the home is essentially proportional. It's not trivial to calculate the saving, given that the heat loss rate varies dynamically over the whole day as the outside temperature varies. The calculation can be done with appropriate math, given some fundamental assumptions, which will be presented shortly. There is also, in our case, a savings from passive solar heating of our home. This raises the temperature above the daytime setpoint of 70°F for much of the photoperiod, obviating the need for furnace heat in the daytime except on the very coldest or cloudiest days. Accounting for solar heating is more complex and I'll defer it until later.

Please note that there is a quick-and-dirty estimate at the end, after all the math, showing quickly that the savings in heat are modest for plausible reductions in the lower temperature setting.

Spoiler alert: the savings from reducing the nighttime low limit from 62°F to 60°F are very small, an estimated 2.5% of total heat demand. There are clear reasons for this that don't apply as strongly in other houses, particularly those with less insulation.

At the end I also make an estimate of where the heat input goes – a tiny bit into warming the air, but most into re-warming the furnishings, walls, etc. – the solid objects that constitute a big thermal mass, much like an adobe wall is touted for.

Here's a **summary of the sections below**, with hyperlinks directly to those sections:



## 1. The system, described

**State variable:**

Interior temperature, $T_{in}$

**Parameters** (fixed values that set the desired behavior):

Daytime interior temperature, $T_{in,hi}$

Nighttime interior low temperature, $T_{in,lo}$

Time that heating starts for the daytime, $t_{start}$ – or, alternatively, the time that attaining $T_{in,hi}$ is desired

Time that heating stops in the late evening, $t_{end}$

Rate constant for heat loss, k, as rate of temperature decline per unit time per unit temperature difference from interior to exterior. ***This is a key assumption, that heat loss is linear in the temperature differential between indoors and outdoors. It would be interesting to check this (and fairly involved!)***

Rate of heat gain from furnace, H, as rate of rise of interior temperature at zero gradient in temperature from interior to exterior

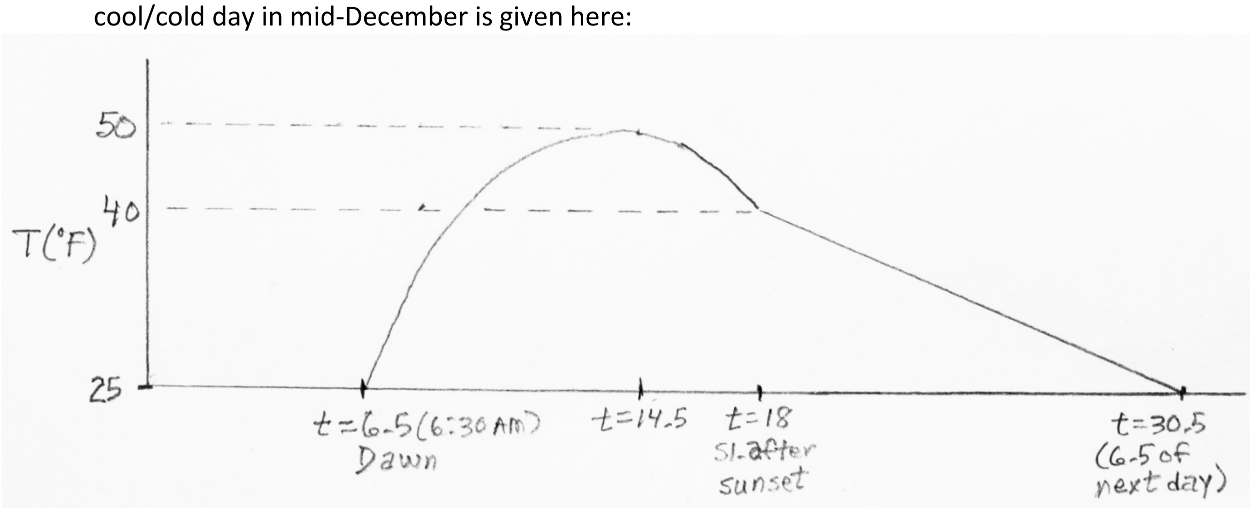
This is taken as a maximal rate, $H_{max}$, for the heating from dawn to the time that $T_{in,hi}$ is first attained.

For other times of day, it is taken as a continuous function sufficient to maintain $T_{in}=T_{in,hi}$

For the nighttime after $T_{in,lo}$ is reached, H is taken as sufficient to maintain $T_{in}=T_{in,lo}$

**Environmental driving variable:**

Exterior temperature, $T_{out}(t)$ as a function of time of day. A sketch of a typical recent sunny but cool/cold day in mid-December is given here:




I break this up into a standard pattern with a number of parameters for two periods:

The photoperiod:

From dawn at $t_{dawn}$, the time of lowest $T_{out}=T_{out,dawn}$, to the end of the photoperiod, $t_{sunset}$. During this time, $T_{out}$ is taken to be a sine function

The peak temperature in midday, $T_{out,peak}$, at the time of the peak, $t_{peak}$

The night, from $t_{sunset}$ to 24 h + $t_{dawn}$ of the next day (hence, the +24 h). During this time the observed pattern is a very closely linear drop in temperature

For the sketched trend, the parameters are:

$t_{dawn}$ = 6.5 (hours). I also use the simpler form, $t_0$.

$T_{out,dawn}$=25°F

$t_{peak}$= 14.5 (2:30 PM); actually, it's a bit later, and the decline in $T_{out}$ after this time is steeper than the sine wave

$T_{out,peak}$= 50°F

$t_{sunset}$= 18 (more like 17.5 = 5:30 PM, really)

Thus, we have in this case

$$T_{out} = 25 + 25\sin\left(\frac{\pi}{2}\frac{(t-6.5)}{8}\right),\ \mathrm{t} \in [6.5,18]$$
$$= 44.3 - 1.54*(t-18),\ \mathrm{t} \in [18,30.5] \tag{1.1}$$

This gives $T_{out}$ = 44.3°F at sunset, which is higher than the observed value of about 40°F. We could use a more accurate function than the sine in further work.

We can write the sine as

$$\sin(c(t-t_0)),\ c = \frac{\pi}{16} = 0.196$$

**Constitutive equation:**

Rate of temperature change = gain from heating minus loss from conduction and convection, taken as proportional to the interior-exterior temperature difference:

$$\frac{dT_{in}}{dt} = H - k[T_{in}(t) - T_{out}(t)]$$
$$= -kT_{in} + [H + kT_{out}(t)] \tag{1.2}$$

This can be solved using the solution of the homogeneous equation,

$$\frac{dT_{in}}{dt} = -kT_{in}$$
$$\rightarrow T_{in}(t) = T_{in}{}^{0}\ \mathrm{e}^{-k(t-t_0)} \tag{1.3}$$
$$T_{in}0 = Tin|_{t=t_0}$$

with the addition of the form for integrating the inhomogeneity, H+k$T_{out}$(t),

$$T_{in}(t) = T_{in}{}^{0}e^{-k(t-t_0)} + \int_{t0}^{t} dt'[H + kT_{out}(t)]e^{-k(t-t')} \tag{1.4}$$

That this is the solution can be verified by differentiation.

This complex equation is needed in only two of the five time periods, which periods will now be described:

**2. Considering different heating and cooling regimes over the day**

**Time periods:**

(1) Initial warming at full heating power, from dawn, $t_{dawn}$, until the daytime comfortable temperature, $T_{in,hi}$, is first reached at time $t_{hi,0}$. The solution will be rather complicated and complex (I use complex variables to do the integration), because $T_{out}$(t) is varying as a sine.

(2) Maintenance of constant comfortable temperature, $T_{in,hi}$, from $t_{hi,0}$ to sunset, during which the outdoor temperature varies as a sine. The differential equation is not needed – one just needs to set the heating rate, H, to get a constant temperature:

$$\frac{dT_{in}}{dt} = 0 = H - k[T_{in,hi} - T_{out}(t)]$$
$$\text{or} \tag{1.5}$$
$$H = k[T_{in,hi} - T_{out}(t)]$$

This is easily solved, since we know the closed form (analytic form) for $T_{out}$(t).

An actual furnace does not alter its output continuously to keep $T_{in}$ constant, as we all know. It cycles on and off; we're only using the average rate here.

(3) Maintenance of constant comfortable temperature from sunset to the next day's dawn, during which $T_{out}(t)$ is a linear function of time. The equation is the same as the last line of Eq. (1.5), with a different form for $T_{out}(t)$

(4) Slow cooling of the home after the furnace shuts off at $t_{end}$, until $T_{in}$ first drops to the desired low point, $T_{in,lo}$, at time $t_{lo,0}$.

(5) Maintenance of the low nighttime interior temperature by adjustable heating. The equation is Eq. (1.5), with $T_{in,hi}$ replaced by $T_{in,lo}$

**3. Bookkeeping – the heating demand:**

Besides tracking $T_{in}(t)$, we want to keep track of the total heating needed,

$$H_{tot} = \int_{t_0}^{t_0+24} dt\, H(t) \qquad (1.6)$$

**4. Estimating numerical values of furnace max. heating, $H_{max}$, and heat-loss coefficient, k:**

This is a cute exercise. I used these observations:

(1) To raise $T_{in}$ from low to high value (8°F) takes 1.5 h in the morning. This is not ascribed to H alone, of course, since heat loss is happening concurrently. We'll make some approximations shortly.

(2) When the furnace shuts off at 10 PM, it stays off for about 6 hours, till 4 AM. We can use the predicted (or observed) $T_{in}$ at 10 PM and at 4 AM to figure the rate of heat loss. The heat loss rate is not constant, since $T_{out}$ is concurrently varying. We'll figure this out shortly, also.

Figuring out H: the solution can be made as complicated as we wish, but a very good approximation is to use the average $T_{in}$ and the average $T_{out}$ , plus the average rate of rise of $T_{in}$, in solving

$$\frac{dT_{in}}{dt} = H - k[\overline{T}_{in} - \overline{T}_{out}] = \frac{8°F}{1.5h} = 5.33 \qquad (1.7)$$

Over the 1.5h warm-up period, $T_{in}$ rises almost linearly from 62°F to 70°F, for an average value of 66°F. Over that same time, $T_{out}$ also rises almost linearly, from 25°F to the value given in Eq. (1.1), or 32.2°F, for an average value of about 28.6°F.

Thus, the average temperature differential from inside to outside is 66-28.6= 37.4°F.

We still need the value of k to proceed:

For the 6 h needed for $T_{in}$ to drop from 70°F to 62°F, the mean $T_{out}$ is the average of $T_{out}$ at 10 PM and at 4 AM. These values are readily calculated as 44.3-1.54*4= 38.1 and 44.3-1.54*10=28.9, for an average $T_{out}$ of 33.5°F.

Thus, we have

$$\begin{aligned} &\frac{dT_{in}}{dt} = k*(66-33.5) = \frac{8}{6} \\ &\rightarrow k = 0.0410,\ \text{in units of h}^{-1} \end{aligned} \qquad (1.8)$$

Back to figuring out the heating rate. In the first time interval with full heat on, we have

$$\begin{aligned} H &= 5.33 + k[\overline{T}in - \overline{T}out] \\ &= 5.33 + 0.0410*37.4 \\ &= 6.86 \end{aligned} \qquad (1.9)$$

**5. The intricate mathematical solution for the warm-up period, coming with a realization about a simpler method:**

We have a formal expression, Eq. (1.4), for the indoor temperature as a function of time with the heat on as the indoor and outdoor temperatures both change. We should verify that it gives the correct answer (indoor temperature reaches 70°F in 1.5 hours, from dawn). Then, we can use it for predicting heating demand with a different setting of the overnight temperature, $T_{in.lo}$, which comes with a different heating time after dawn.

The homogeneous term, $T_{in}^0$exp(-k*(t-$t_{dawn}$)), is easy to calculate, now that we figured out the numerical value of k.

The formal integration of the inhomogeneous term has a bit of interesting math, so we'll do this now. We'll split up this term into one that is constant over time, involving H+ka in the integrand, and one that has the time-varying part, kb sin(c(t-$t_0$)).

The first term is

$$\begin{aligned}(H+ka)\int_{t0}^{t} dt' e^{-k(t-t')} &= (H+ka)e^{-kt}e^{kt'}\Big|_{t_0}^{t} \\ &= (H+ka)e^{-kt}\frac{e^{kt}-e^{kt_0}}{k} \\ &= \left(\frac{H}{k}+a\right)[1-e^{-k(t-t_0)}]\end{aligned} \quad (1.10)$$

This looks just like the solution for a transient, which ultimately reaches the steady value of H/k+a – which is a rather large number, 6.86/0.0410+25 = 192 -the temperature if the heat stayed on (ignoring some variation in outside $T_{out}$)!

The second term, for the sinusoidally varying part of the outdoor temperature, is the complex part – literally so, deriving the form using complex variables. it involves the integral

$$I = \int dt' e^{kt'} \sin(c(t'-t_0)) \quad (1.11)$$

We can write the sine as its form in complex variables, (1/2i)($e^{ict'}$-$e^{-ict'}$), to get

$$\begin{aligned}I &= \frac{1}{2i}\int dt' e^{kt'}(e^{ic(t'-t_0)} - e^{-ic(t'-t_0)}) \\ &= \frac{1}{2i}\left(\frac{e^{(k+ic)x}}{(k+ic)} - \frac{e^{(k-ic)x}}{(k-ic)}\right), \text{ writing t-t}_0\text{=x} \\ &= \frac{1}{2i}\left(\frac{(k-ic)e^{(k+ic)x}-(k+ic)e^{(k-ic)x}}{(k^2+c^2)}\right) \\ &= \frac{e^{kt'}}{(k^2+c^2)}\left[\frac{k(e^{icx}-e^{-icx})}{2i} - ic\frac{(e^{icx}+e^{-icx})}{2i}\right] \\ &= \frac{e^{kt'}}{(k^2+c^2)}[k\sin(c(t-t_0)) - c\cos(c(t-t_0))]\end{aligned} \quad (1.12)$$

to be evaluated at the upper and lower limits, t and $t_0$.

I checked this vs. the classic reference, Gradshteyn and Ryzhik.

Using the full expression for the homogeneous and inhomogeneous terms, I evaluated $T_{in}$ at t=8. I obtained 69.94°F, so the formula is good.

There is an obvious simplification. The complicated formula gives virtually the same answer as if we took the rate of temperature rise, $dT_{in}/dt$, as a constant over the interval t=6.5 to t=8, using the mean values of indoor and outdoor temperatures; the sinusoid is nearly linear in time, anyway. So, we can use the much simpler approximation, Eq. (1.7)!

We can do this also for the cooling-down period with the heat off.

**6. Solutions for heat use in all 5 time intervals, using simpler yet accurate math:**

(1) Warm-up. Use Eq. (1.7), evaluating mean indoor and outdoor temperatures during this period. This is handy when we consider a modification for lower nighttime setting of $T_{in}$. In this case, we might want to start heating before dawn so that $T_{in}$ reaches our desired point of 70°F by 8 AM again. The result is the heating effort, $(t-t_0)*H_{max}$, to put into our calculation of accumulated heating effort, Eq. (1.6)

For the values chosen at first (i.e., start heating at 6:30 AM and hit desired 70°F at 8 AM), the result is 1.5 hours of heating at a rate H=6.86, or 10.29 units (conversion to joules or the like can be done, with some work; our furnace has a nominal heat input of 80,000 Btu/h = 84.4 MJ/h or 23.4 kW; this is the equivalent of 6.86 in our abstract units).

(2)+(3) Daytime constancy of $T_{in}$. We use Eq. (1.5) to solve for H(t) and integrate this as Eq. (1.6) from time $t_{hi,0}$ (end of the warmup period) to time $t_{end}$, when the thermostat is set low and $T_{in}$ is allowed to drift downward toward $T_{in,lo}$.

We need to do the integral

$$Htot = \int_{t_{hi,0}}^{t_{end}} dt' k(T_{in,hi} - T_{out}(t')) \qquad (1.13)$$

Let's omit the k and add it at the end.

$$H_{tot} / k = \int_{t_{hi,0}}^{t_{end}} dt' T_{in,hi} - \int_{t_{hi,0}}^{t_{sunset}} dt'[a + b\sin(c(t-t_0)) - \int_{t_{sunset}}^{t_{end}} dt'[A - B(t - t_{sunset})] \qquad (1.14)$$

Here, we had to break up the integration over $T_{out}(t)$ into the two periods, one with sinusoidal variation and the other with linear variation. For the latter, I coded A for 44.3°F and B for 1.54°F/h, as calculated earlier.

We can leave all the parameters in general form, allowing for different times of the winter, different setpoints, etc. I leave this for the reader to do.

We have to do the integral

$$\int_{t_{hi,0}}^{t_{sunset}} dt' \sin(c(t-t_0)) = \frac{1}{c}[\cos(c(t_{hi,0} - t_0)) - \cos(c(t_{sunset} - t_0))] \qquad (1.15)$$

For our case, going term-by-term:

$$H_{tot} / k = 14*70 - 10*25 - \frac{25}{0.196}(0.957 - (-0.631)) - 44.3*4 + 1.54*\left(\frac{4^2 - 0}{2}\right) \qquad (1.16)$$

This is 362.5, yielding $H_{tot}$=14.86. Thus, daytime heating takes a bit more than warmup.

(4) Cool-down. No heating, but we need to figure out how long this period lasts. We already observed this as 6 h, so that it ends at 4 AM the next day.

(5) Maintaining $T_{in}$ at $T_{in,lo}$ until dawn, using heat. The average heating rate is k multiplied by the average difference between $T_{in}$ and $T_{out}$. The average of $T_{in}$ is trivially $T_{in,lo}$ = 62°F. The average $T_{out}(t)$ is halfway between its value at 4 AM and its value at 6:30 AM, or 25+0.5*1.54*2.5=26.9°F. The integrated heating is then the time interval, 2.5h, multiplied by k and by (62-26.9), or 3.60 units.

The grand total heating is then 10.29 + 14.86 + 0 + 3.60 = 28.75, of which 36% is spent in warming up at dawn.

**7. The ultimate question (of Life, the Universe, and Everything): how much energy is saved by setting the nighttime low temperature lower?:**

Let's increase the gap from high to low by 50%, or to 12°F. ***A drop in the indoor temperature maintained at night should decrease the mean differential between indoor and outdoor temperatures, saving energy.*** The new $T_{iin,lo}$ becomes 58°F. Can our well-insulated house cool this far down in these conditions? It only drops 8°F in 6 h, and there's only 2.5 h more till heating would start (less, in fact: heating has to start earlier). It can't work, so let's set a lesser goal, $T_{in,lo}$=60°F. Let's estimate when this might be reached. First, let's estimate the time that warming up must start. The new T rise is 10°F, or 10/8 larger than the original 8°F. Thus, it might take 10/8 longer (not exactly; average $T_{in}$ is lower but average $T_{out}$ is a bit lower, also). We estimate a heating time of (10/8)*1.5h, or 1.88h.

Is there enough time to cool down to this more modest setting of 60°F? Let's calculate when $T_{in}$ drops to 60°F. Let this time be called $t^*$. We can write that, during cooling down,

$$T_{in} = 70 - k(\overline{T}_{in} - \overline{T}_{out})*(t*-22) = 60 \qquad (1.17)$$

Now, average $T_{out}$ has to be calculated. It is the temperature at 10 PM (t=22), minus the drop from that time to $t=t^*$. Both are calculated from the slope, 1.54°F per hour. Skipping the obvious algebra, we have

$$\begin{aligned}\overline{T}_{out} &= [38.1 + 38.1 - 1.54*(t*-22)]/2 \\ &= 38.1 - 0.77*(t*-22)\end{aligned} \qquad (1.18)$$

Our equation is then

$$k(\overline{T}in - \overline{T}out) = \frac{70-60}{x} = k*[65-(38.1-0.77*x)] \qquad (1.19)$$

where I have written $x=t^*-22$. Multiplying the final two expressions by x, we get a quadratic,

$$10 = 26.9kx + 0.88kx^2 \qquad (1.20)$$

We can rearrange this to standard form, $a+bx+cx^2$ and then solve, obtaining x=7.47 hours. This puts the cooling down to 60°F at t=22+7.47 = 29.47, or 5.47 hours into the new morning. Yes, this is before we need to turn on the heater to warm back up to 70°F.

Two calculations remain:

(1) When do we need to turn on the heat again, so that we reach 70°F by 8 AM, as before? The estimate in the first paragraph of this section puts it 0.38h earlier, at t=6.12. I did a better numerical estimate, finding a tiny shift, to t=6.13. I did this by calculating what starting time gives 62°F at t=6.5, the same as in the original case. Note that this means the total heating during warmup is now 6.86, the furnace rate, multiplied by the total time, now 1.5+0.37 = 1.87h, or 12.83 units.

(2) How much heat is used between the cool-down time, t=29.47 or 5.47 of the next day, and t=6.13, when warm-up starts? This heat has to balance the integrated loss, which is k times the average temperature differential. We have $T_{in}$ stabilized at 60°F. Using the linear Eq. (1.1), we find that

$$\overline{T}out = 0.5*(26.6+25.6) = 26.1 \qquad (1.21)$$

We then have the mean temperature difference as 33.9°F. This gives a heat loss rate of 0.041*33.9 = 1.39. It persists for 0.26h, giving us a heating demand of 0.36 units.

The new total heating demand is:
12.83 units in the first interval, warm-up
14.86 units in the second and third intervals, maintaining $T_{in}$ at 70°F
0 units in the fourth interval, cooling down
0.36 units in the fifth interval, maintaining $T_{in}$ at 60°F until warm-up starts
This is a total of 28.05 units.
**The saving is minuscule, 0.7 units out of 28.75 units, or 2.5%!**
**The real savings would be in turning down the daytime temperature!**
The reason that the savings are so small is that there is only a short time before dawn that the indoor temperature is allowed to drop below the original setpoint of 62°F. This, in turn, results from the good insulation of our home and the modest outdoor temperatures. A home with less insulation would show greater gains from turning down the temperature at night...but only in a relative sense; the total heating demand would scale almost linearly with the heat loss coefficient, k.

**Much greater savings yet come from having passive solar heating.**
We have passive solar heating. In our home, on a normal sunny winter day, we need minimal heating in the daytime, perhaps 0.5 hour total in the hour after we attain 70°F for the first time at the end of warm-up. After that, $T_{in}$ actually rises above 70°F, to about 75°F at the peak, and we hit 70°F right about at 10 PM. The daytime heat is then 6.86*0.5 = 3.43 units, vs. 14.86 units, a saving of 13.43 units, 47% of the total heating!

**8. Another analysis: where did the heat go during warm-up?:**
The furnace heats the air, but the air then transfers heat to the structure and furnishings in the home, which also cooled down overnight. How much heat went into the air vs. the "solids," as we may call them?
A fair approximation can be derived from knowing how much heat the furnace transferred to thehome interior in the 1.5h warm-up period and then compare it to the total heat content added to the air in that time.
Heat transferred to the home by the furnace: a quick calculation assumes that the rating of the furnace is accurate. The certification says the heat input rate is 80,000 Btu/h, or 84 MJ/h. In 1.5h, the input is 126 MJ. Not all of it gets captured by the heat exchanger. I'll assume 80% efficiency, so that about 101 MJ get circulated into the home, the rest going up the exhaust.
Heat content added to the air inside the home: the floor area of the heated section of our home is about 1800 $ft^2$ or 167 $m^2$. The mean height of the ceiling is about 8' or 2.44 m. Ignoring the volume of walls and furnishing, this gives us an air volume of 408 $m^3$. The heat added to the air is its volume multiplied by the heat capacity per volume and by the temperature rise. The heat capacity per volume is the molar density of air, taken at the mean temperature of the air and our local atmospheric pressure, multiplied by the heat capacity per mole at constant pressure. Note that there is a small ejection of heated air as the air expands upon heating. The molar volume rises by the fraction of change in absolute temperature, about 1.5%. I'll ignore this. Back to the molar density of air: this is calculated from the ideal gas law:

$$PV = nRT$$
$$\rightarrow \frac{n}{V} = \frac{P}{RT}$$
$$= \frac{88500 Pa}{(8.314 J\, mol^{-1} K^{-1})(292 K)} \qquad (1.22)$$
$$= 36.4\, mol\, m^{-3}$$

Here, all in SI (metric) units, P is the atmospheric pressure, V is the volume, n is the number of moles of gas, R is the universal gas constant, and T is the absolute temperature. I used a mean temperature of 66°F or 18.9°C.

Continuing, we have the number of moles of air in the air space of our home as

$$n = 36.4\, mol\, m^{-3}\, x\, 408\, mol$$
$$= 14,850\, mol \qquad (1.23)$$

We need to multiply this by the heat capacity of air per mole at constant pressure, 29 J $mol^{-1}K^{-1}$, and by the temperature rise, 8°F or 4.4°C. We get the change in heat content as

$$\Delta Q = (14,850\, mol)(29\, J\, mol^{-1} K^{-1})(4.4 K)$$
$$= 1.89 x 106\, J = 1.89\, MJ \qquad (1.24)$$

This is very small compared to the total heat put out by the furnace; it is only 1.9%!

We can estimate how much solid material is being heated up, absorbing the rest of the heat. Of course, we have to subtract the heat lost to the outdoors, first, which is the loss rate, k multiplied by the mean temperature differential, multiplied in turn by the duration of heating:

$$\Delta Q_{out} = k(\overline{T}_{in} - \overline{T}_{out})(1.5h)$$
$$= 0.041 * 37.4 * 1.5 \qquad (1.25)$$
$$= 2.30, \text{ in our abstract energy units}$$

We can use the conversion that 6.86 abstract energy units (heat output of the furnace in one hour) is 80% of 84 MJ, so that one abstract unit is 9.8 MJ. This gives us a heat loss of 22.5 MJ...again, much larger than the heat added to the air. This gives us an amount of heat, (101-1.9-22.5) MJ = 76.6 MJ to assume as going into heating the solid material in the home.

We can take a mean heat capacity of solids as 4 J per gram per K, or 4 kJ per kg per K. We come up with a quantity of 19,200 kg *(rise in temperature in K)! If we take the rise in temperature as the same as for the air, we get a heated mass of about 4300 kg or 4.3 metric tonnes. Is this reasonable? How much thickness of material is this, around the area of walls, floor, and ceiling? I omit for simplicity the furnishings sitting away from the walls.

The linear extent of walls is about 400 feet or 122 m. At a mean height of 2.44 m, this gives a wall area of 297 $m^2$. The floor and ceiling areas combine as twice the ground area of 1800 $ft^2$ or another 816 $m^2$, giving us a total of about 1113 $m^2$. To get a mass of 4300 kg over that area at a mean density of 1.5 tonnes per $m^3$ (1.5 times the density of water) requires a thickness of 2.6 mm, or about 0.1"! Even allowing for the profile of temperature not being sharp but distributed with depth, the penetration depth is only about 5 mm. We're heating a thin skin of the solids. Does this agree with the thermal diffusivity of the solids and the heat-transfer capability of slowly moving air? This is an exercise for another time.

**9. Before I finished the math above: Quick-and-dirty calculation, without the complex math:**

What we can achieve by lowering the nighttime thermostat setting, $T_{in,lo,}$ is a reduction in the mean temperature differential at night between indoors and outdoors.

Let's say we reduce $T_{in,lo}$ by 4°F, from 62°F to 58°F.

Before we did this, we had at night an average indoor temperature a bit closer to $T_{in,lo}$ than to $T_{in,hi}$ – it took 6 of the 8.5 h of the night to drop to $T_{in,lo}$. Let's say the average is about 70% of the way toward $T_{in,lo}$, or about 64.4°F.

The average nighttime outdoor temperature is readily estimated from the second line in Eq. (1.1), as 31.6°F.

Thus, the average temperature differential is about 32.8°F.

If we drop $T_{in,lo}$ by 2°F, we have decreased the differential by 2°F, or about 6%.

This is about the fraction that nighttime heating demand is decreased.

However, the daytime heating demand can't be neglected. The daytime setting lasts 15.5 h. The average $T_{out}$ is higher in the day, but so, too, is $T_{in}$. Without solar heating during the day, the average heating rate in the day is fairly close to the mean rate at night. So, the nighttime heating may be only about 35-40% of the total heating demand. Thus, saving 25% of even 40% is only saving about 5%!