internal_pressure_to_burst_aluminum_can.docx ~/math 21 Apr. 13

How much pressure is in the can when freezing causes it to burst?

The hoop stress is the force exerted circumferentially (perpendicular both to the axis and to the radius of the object) in both directions on every particle in the cylinder wall. It can be described as:

$$\sigma_\theta = \frac{F}{tl}$$

where:

- *F* is the force exerted circumferentially on an area of the cylinder wall that has the following two lengths as sides:
- *t* is the radial thickness of the cylinder
- *l* is the axial length of the cylinder

$$\sigma_\theta = \frac{Pr}{t}$$ (for a cylinder)

Yield strength of aluminum alloys – for inexpensive ones, probably 30,000 psi (http://www.eaglesteel.com/download/techdocs/Aluminum.pdf)

Solve for pressure that bursts the can.

Get the thickness. From http://www.airliners.net/aviation-forums/non_aviation/read.main/977127/, one person measured 0.0038." I could also figure it out from one quote of the empty mass being 15 g → I got 13 g.

For a height of 12 cm and diameter of 6.4 cm (my measts., again), the surface area is:

- Along the length: $2\pi RL = \pi DL = 241\ cm^2$
- On the two ends: $2*\pi R^2 = \pi D^2/2 = 64\ cm^2$
- Total: $305\ cm^2$

Assuming that the thickness is similar all over (a weak assumption), the mass, m, is then ρtA, with the density ρ as $2.7\ g\ cm^{-3}$. Thus

$t = m/(\rho A) = 13\ g/\ (2.7\ g\ cm^{-3}\ 305\ cm^2) = 0.0158\ cm = 0.158\ mm$

Let's convert yield strength to metric units:

$$lb(force)x\frac{kg}{2.2lb}x9.81ms^{-2} = 4.46N$$ (agrees with a Website)

Then,

$$psi = \frac{lb(force)}{in^2} = \frac{4.46N}{in^2}x\left(\frac{in}{2.54cm}\right)^2 = 0.69N\ cm^2 = 6900Pa$$

Of course; 14.7 psi = 101,300 Pa

OK. Then,

$$\text{Pressure} = P = \sigma \frac{t}{R} = 33,000\,psi \frac{6900\,Pa}{psi} \frac{1.58x10^{-4}\,m}{6.4x10^{-2}\,m} = 562,000\,Pa = 5.54\,atm$$

That's not very much overpressure, is it!

If the yield strength is higher…
According to
http://www.aluminum.org/AM/Template.cfm?Template=/CM/ContentDisplay.cfm&ContentID=31517&FusePre, the aluminum in beverage cans is type 3004 or 3104 and has an ultimate yield strength of about 42,000 psi. I should multiply the result above by 1.4, to get about 787 kPa or 7.8 atm.

> [Beverage can bodies are typically 3004 or 3104 in the H19 temper. The 3004-H19 has a typical tensile yield strength of 41.3 ksi, a typical ultimate tensile strength of 42.8 ksi, and an elastic modulus of 10.0 Msi. The ends are typically 5182-H19. The 5182-H19 has a typical tensile yield strength of 57.3 ksi, a typical ultimate tensile strength of 60.9 ksi and an elastic modulus of 10.1 Msi.]

I have not (yet) gone on to compute the energy released on splitting – I'd need to know the deformation amounts. I have also not figured out the temperature at which freezing begins and generates the needed pressure, nor the fraction frozen at rupture. For the last item, I'd need to use the Young's modulus. Oops – maybe not – I didn't count on the phase separation of CO_2 from water as the soda freezes:

From http://www.lifeslittlemysteries.com/2846-frozen-soda-explosion.html

Attempts to speed-cool a soda often result in a busted hull of aluminum and a wall-to-wall coat of sticky slush on the inside of the freezer. A popular belief is that this is because water in the soda expands when it freezes, exceeds the can's carrying capacity, and blows it open.

But that's not the whole story. The boy would not have been hurt if the can had been filled with water alone.

"What's dangerous is the carbon dioxide gas," said Louis Bloomfield, a physicist at the University of Virginia. "Once it has been forced out of the water as the water crystallizes, the carbon dioxide accumulates in the small remaining space in the can and the pressure of that gas skyrockets."

The expansion of water as it freezes is important to soda detonation not because it puts pressure on the can but because it puts pressure on an increasingly cramped reservoir of gaseous C02, which can change volume more radically and store far more energy than pressurized ice.

But if the water had already frozen and created a pocket of pent-up CO2 capable of rupturing the can, why hadn't the Chinese boy's soda already exploded on its own in the freezer?

"The can was stressed to its limit and when he introduced new stresses while trying to pop the top, he evidently found the weak point," Bloomfield told Life's Little Mysteries. "Whereas many cans with tiny defects would already have burst at that point, this particular can was holding on especially long. It might have held out forever, had the boy not handled it."

Though Bloomfield guesses that the boy was "seriously unlucky," the clear lesson of his misfortune for the cautious and occasionally absentminded is to give sodas time to cool in the fridge.

Either that, or switch to canned tomato juice, which won't blow up in your face even if you freeze it.